Eduquas - H - June 3rd '24 - Component 2 (calc)

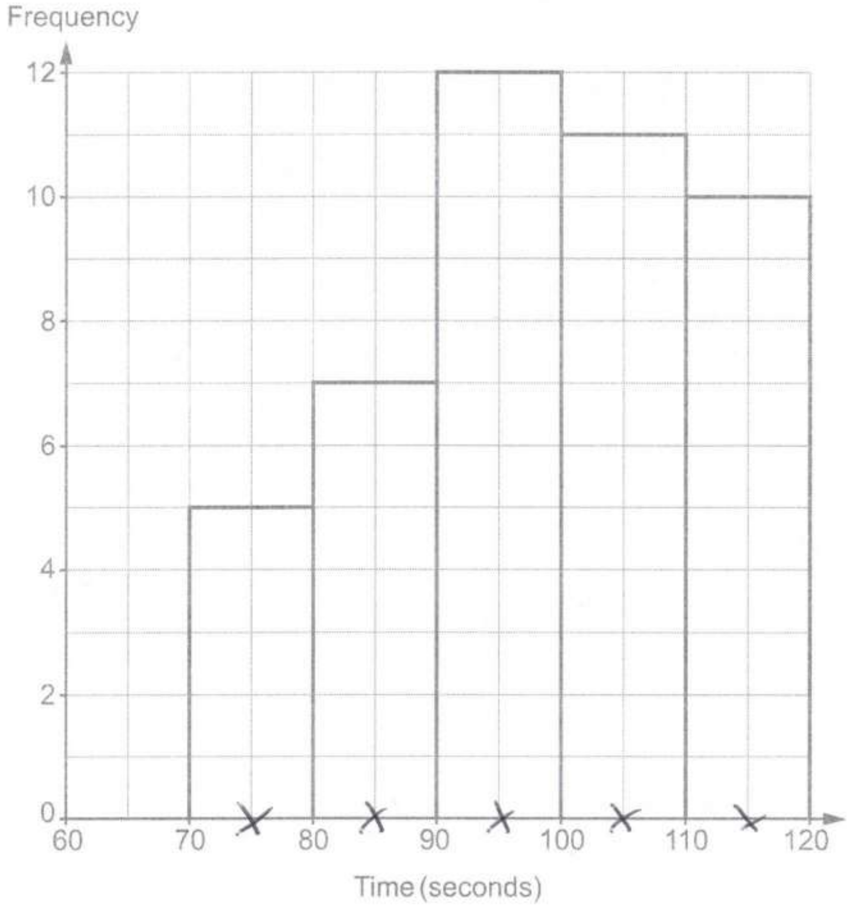
Examiner only

Answer **all** questions.

1. 45 students took part in a 400 m race.
Their times, in seconds, were recorded.
The frequency diagram below shows the results.

The groups used are as follows:

70 ⩽ time < 80, 80 ⩽ time < 90, 90 ⩽ time < 100, 100 ⩽ time < 110 and 110 ⩽ time < 120.



Calculate an estimate for the mean time it took to complete the race. [5]

$$\left.\begin{array}{l} 75 \times 5 = 375 \\ 85 \times 7 = 595 \\ 95 \times 12 = 1140 \\ 105 \times 11 = 1155 \\ 115 \times 10 = 1150 \end{array}\right\} = 4415$$

$\div 45$

$= \underline{98.1}$

 (C300UB0-1)

Turn over.

Examiner only

2. (a) Factorise $x^2 - 5x - 24$. [2]

$+$ $\times$

$(x-8)(x+3)$

(b) Simplify $\left(3h^2\right)^3$. [2]

$3^3 = 27 \quad h^{2\times 3} \quad = \underline{\underline{27h^6}}$

(c) Represent the inequality $-2 < x \leqslant 3$ on the number line below. [1]

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3.

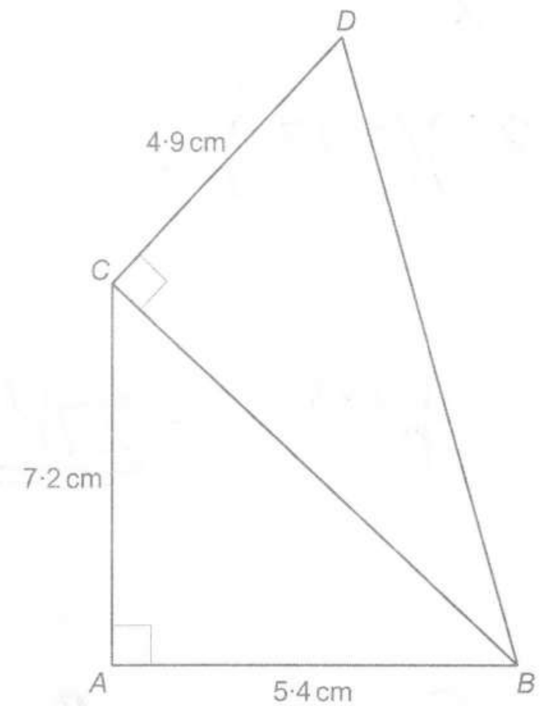

Diagram not drawn to scale

ABC and *BCD* are right-angled triangles.
AB = 5·4 cm, *AC* = 7·2 cm and *CD* = 4·9 cm.

Calculate the area of triangle *BCD*. [5]

$CB = \sqrt{7{\cdot}2^2 + 5{\cdot}4^2} = 9$

Area BCD $= \frac{1}{2} \times 9 \times 4{\cdot}9$

Area of triangle *BCD* is 22·05 cm^2

 (C300UB0-1)

Turn over.

C300UB01 05

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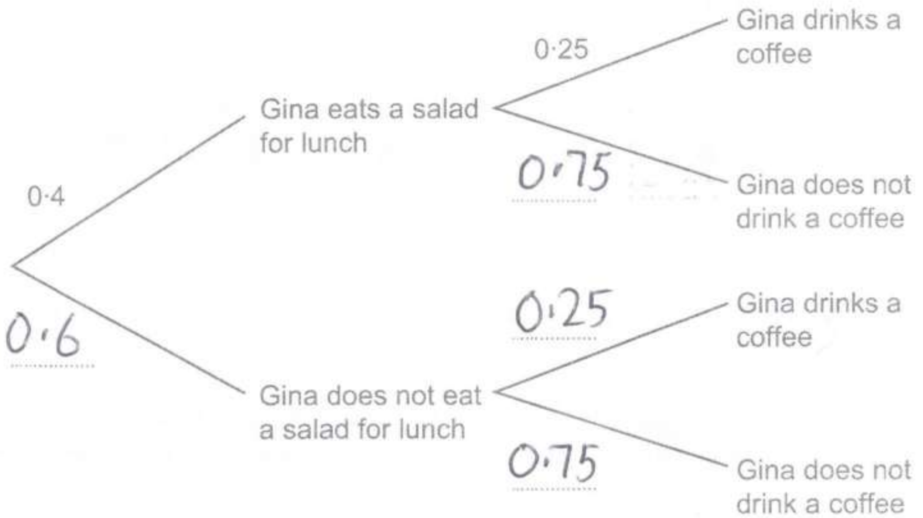
4. For her lunch, Gina sometimes eats a salad and sometimes drinks a coffee.
The probability she eats a salad for lunch on a given day is 0·4.
The probability she drinks a coffee on a given day is 0·25.
Eating a salad for lunch and drinking a coffee are independent.

(a) Complete the following tree diagram. [2]



(b) Calculate the probability that, on a given day, Gina does not eat a salad for lunch but does drink a coffee. [2]

$0{\cdot}6 \times 0{\cdot}25 \qquad = 0{\cdot}15$

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5. When a ball is dropped, the maximum height it reaches after each bounce decreases.

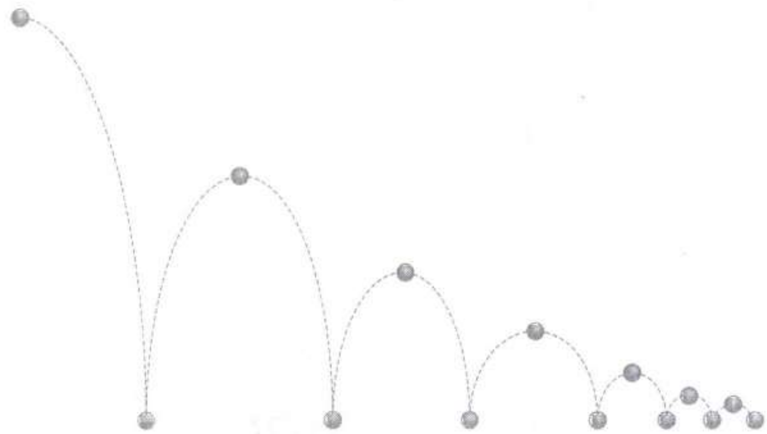

Diagram not drawn to scale

Maddox says,

> Each time the ball bounces it reaches a height that is 55% of the maximum height reached on its previous bounce.

(a) The ball is dropped from a height of 4 m.

What height will the ball reach above the ground after its 7th bounce? [3]

$4 \times 0.55^7 = 0.060897\ldots$

$= 0.0609\text{m}$

(b) (i) State any assumptions you have made in answering part (a). [1]

Each bounce is on the same surface / is just as bouncy

(ii) What effect could your assumption have on your answer to part (a)? [1]

Harder surface could bounce higher, softer may bounce less

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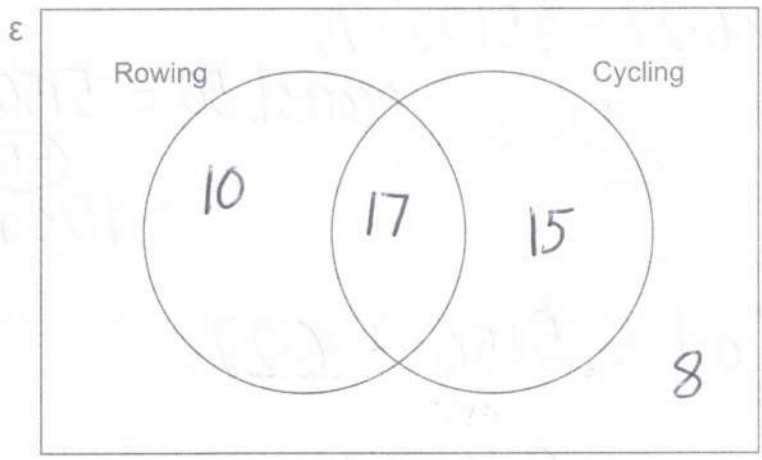
6. There are 50 people in a sports club.
Two of the sports the people can take part in are rowing and cycling.

Of these 50 people:

- 27 take part in rowing
- 32 take part in cycling
- 8 do not take part in rowing or cycling.

(a) Complete the Venn diagram to show this information. [2]



(b) One person is chosen at random from the sports club.
Find the probability that they take part in rowing but not in cycling. [2]

$\frac{10}{50}$

Examiner only

7. Abdul is travelling from the UK to Brazil for a holiday.
Abdul has £825 to exchange for Brazilian reals (R$).
The exchange rate is £1 = 6.27R$.
The bank Abdul uses to exchange his money only has 50R$ notes.

What is the maximum amount of Brazilian reals Abdul can buy and what will this cost him in pounds?
Give your answer correct to the nearest penny.
You must show all your working. [5]

£825 x 6·27 = £5172·75

nearest 50 = 5150

÷50

= 103 notes

Cost = 5150 ÷ 6·27

= 821·371...

= £821·37

8. Make R the subject of the formula $P = \sqrt[3]{RQ}$. [2]

$P^3 = RQ$

$\frac{P^3}{Q} = R$

Examiner only

9. (a) Show that $2x(7x-8y)-3y(x-5y)\equiv 14x^2-19xy+15y^2$. [2]

$= 14x^2 - 16xy - 3xy + 15y^2$

$= 14x^2 - 19xy + 15y^2$

(b) In part (a) the identity symbol ($\equiv$) has been used.
Explain why this has been used and not an equal sign. [1]

It is always true/

It is true for all values of x and y

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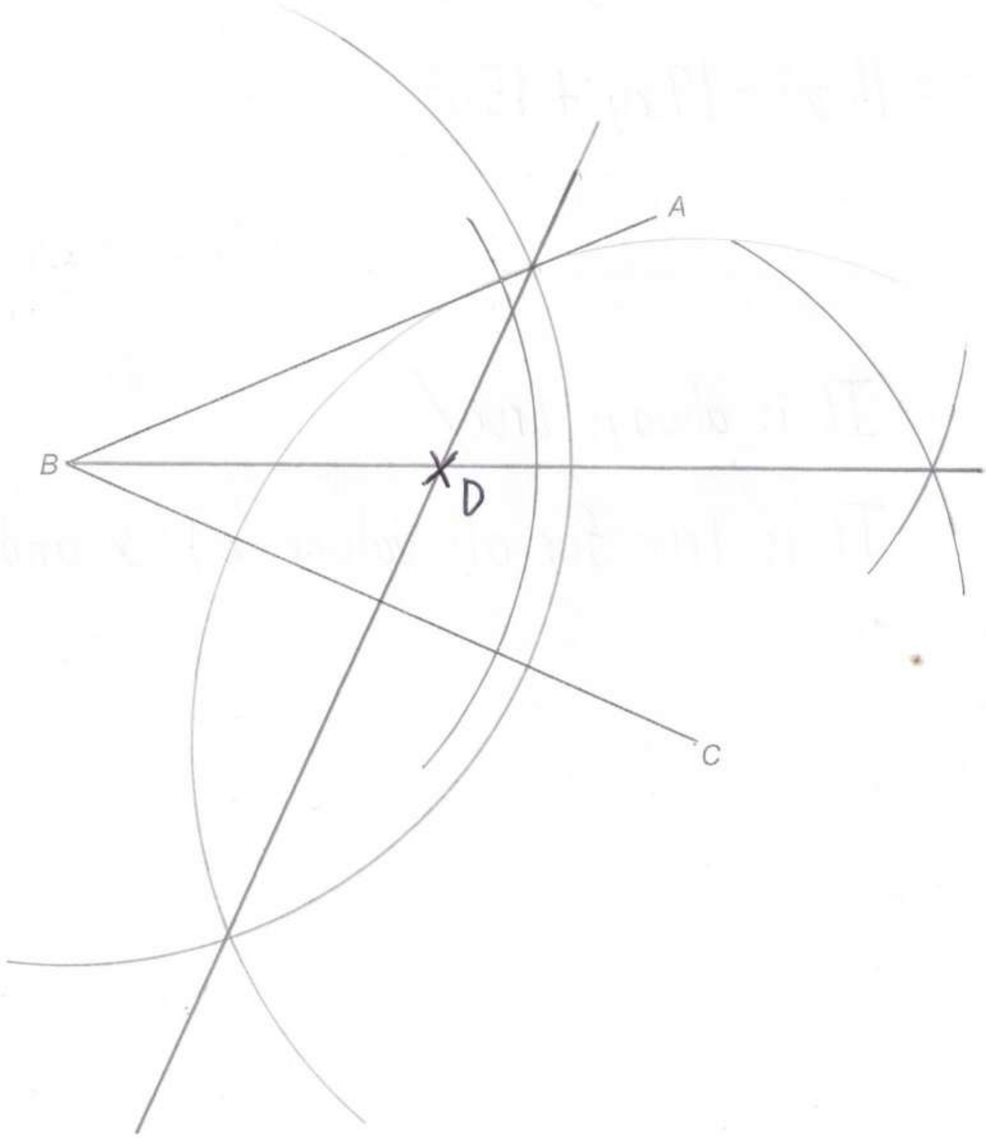
10. The point D is equidistant from the lines AB and BC.
Point D is also equidistant from the points B and C.

Using only a ruler and a pair of compasses, **construct** suitable arcs and lines to show the position of point D. [4]



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11.

Use:

$$\text{Pressure} = \frac{\text{Force(N)}}{\text{Area}(\text{cm}^2)}$$

A cylindrical fish tank stands on horizontal ground.
The base of the fish tank has a radius of 24 cm.
All the base of the fish tank is in contact with the ground.

The fish tank exerts a pressure of 0·97 N/cm^2 on the ground.
What force is exerted on the ground by the fish tank? [4]

$F = P \times A$

$= 0{\cdot}97 \times \pi \times 24^2$

$= 1755{\cdot}27...$

Force = 1755 N

 (C300UB0-1)

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12. The diagram below shows a vertical cliff which is 35 metres high.
Pierre is standing at point P.
From point P, the angle of elevation to the top of the cliff is 72°.

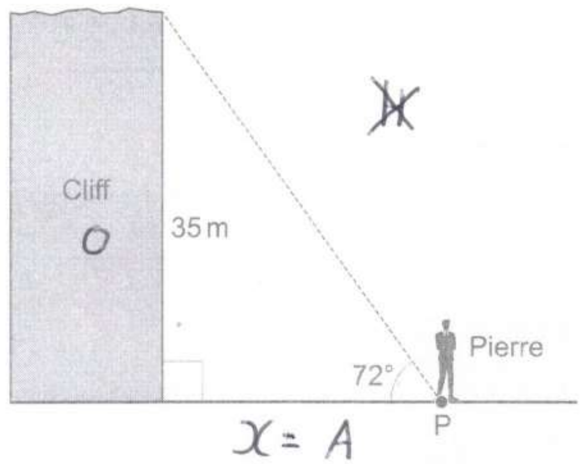


Diagram not drawn to scale

Pierre sees this warning sign.

Warning!
Coastal Erosion.
It is not safe to stand within 10 m of the base of the cliff.

Is Pierre standing a safe distance from the base of the cliff? [4]

Yes ✓ No ☐

Show how you decide.

$T\frac{O}{A}$ $\quad x = \frac{O}{T} = \frac{35}{\tan 72}$

$= 11.37\ldots \text{ m}$

Turn over.

Examiner only

13. A solution to the equation $2x^3 - 5x - 23 = 0$ lies between 2 and 3.
Use the method of trial and improvement to find the solution correct to 1 decimal place. [4]

x

$2.5 \quad (2\times2.5^3)-(5\times2.5)-23 = -4.25$

$2.7 \quad = 2.866$

$2.6 \quad = -0.848$

$2.65 \quad = 0.969$

so $2.6 < x < 2.65$ $\quad x = 2.6$ (1dp)

14. Solve $\frac{4x+5}{2} - \frac{x+1}{3} = \frac{5}{6}$. $\times 6$
You must show all your working. [4]

$3(4x+5) - 2(x+1) = 5$

$12x + 15 - 2x - 2 = 5$

$10x + 13 = 5$

$10x = -8$

$x = -0.8$

Examiner only

15. A solid cone has a base radius of y cm.
The height of the cone is six times its radius. $\rightarrow 6y$
The cone is made from material which has a density of $7\,g/cm^3$.
Write an expression for the mass of the cone in grams.
Give your answer, in terms of y and π, in its simplest form. [3]

$$Vol = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times y^2 \times 6y = 2\pi y^3$$

M

D V

$$M = D \times V$$

$$= 7 \times 2\pi y^3$$

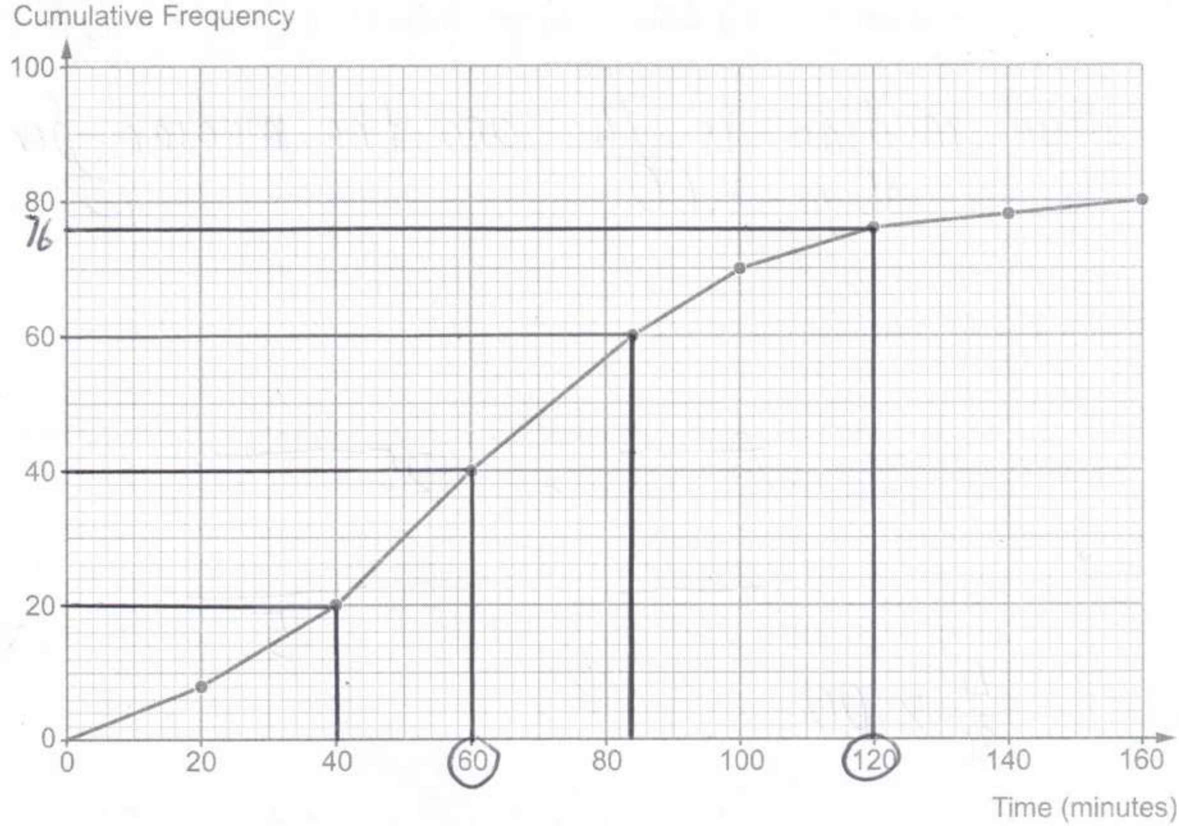
$$= \underline{\underline{14\pi y^3}}$$

C300UB01
15

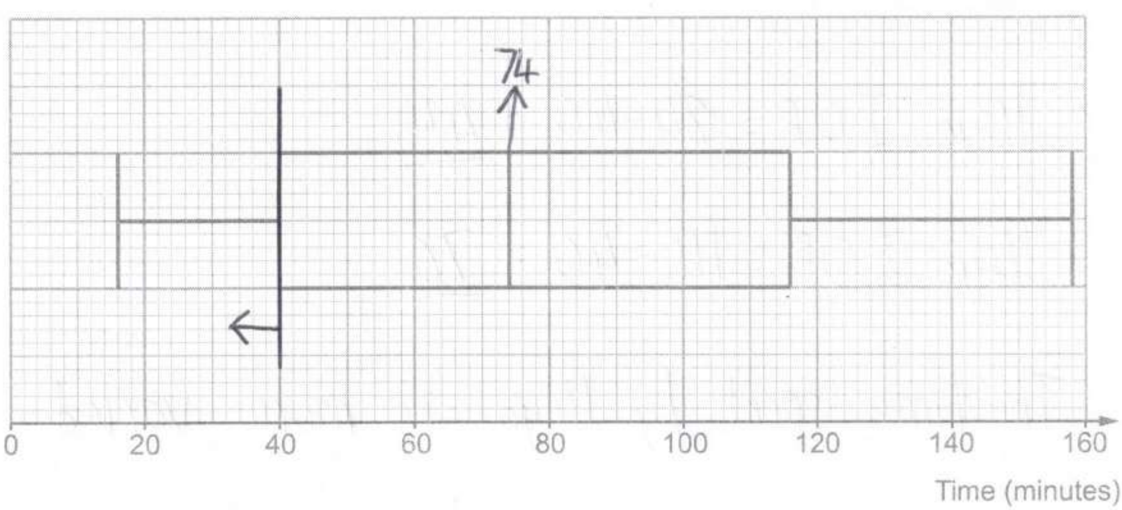
16. A video game company investigated how long, in minutes, one group of 10-year-old students and one group of 14-year-old students spent playing video games last Wednesday evening.

Diagrams displaying the results for each age group are shown below.

10-year-old students



14-year-old students



 (C300UB0-1)

Examiner only

(a) Lewis says,

"14-year-olds spend longer, on average, playing video games than 10-year-olds."

Is Lewis correct?

Yes ✓ No ☐

Use information from the diagrams to justify your answer. [1]

14y.o median is 74 and the median for 10yo = 60

(b) Complete the following sentences using information from the diagrams.

(i) The percentage of **14-year-old** students that spent less than 40 minutes playing video games last Wednesday evening was 25 %. [1]

(ii) The percentage of **10-year-old** students that spent more than 120 minutes playing video games last Wednesday evening was 5 %. [2]

$\frac{4}{80} \times 100$

(c) Compare the inter-quartile range of the times each age group spent playing video games last Wednesday evening.
What conclusions can you draw about the length of time 10-year-olds and 14-year-olds spent playing video games last Wednesday evening? [2]

10yo IQR = 84 − 40 = 44

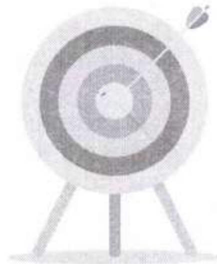
14yo IQR = 116 − 40 = 76

The time spent by 14yo is more varied

 (C300UB0-1) Turn over.

Examiner only

17. Noah and Jenny play a game.
Each shoots an arrow at a target.
The arrow is removed from the target after each shot.



For each of their shots:

- Noah hits the target with a probability of $\frac{1}{5}$.
- Jenny hits the target with a probability of $\frac{3}{10}$.

Noah and Jenny take it in turns to shoot an arrow at the target.
In the game, Noah shoots the first arrow.
When one of them hits the target the game ends and that person wins the game.

(a) Calculate the probability that Jenny wins the game on her first shot. [2]

$\frac{4}{5} \times \frac{3}{10} = \frac{6}{25}$

(b) Calculate the probability that Noah wins the game on his second shot. [2]

$\frac{4}{5} \times \frac{7}{10} \times \frac{1}{5} = \frac{14}{125}$

 (C300UB0-1)

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18. Kian went for a run.
The route he runs is 3000 metres, **correct to the nearest 100 metres**.
He ran the route in a time of 720 seconds, **correct to the nearest 10 seconds**.

Calculate Kian's **greatest** possible average speed for this run.
Give your answer in metres per second.
You must show all your working. [3]

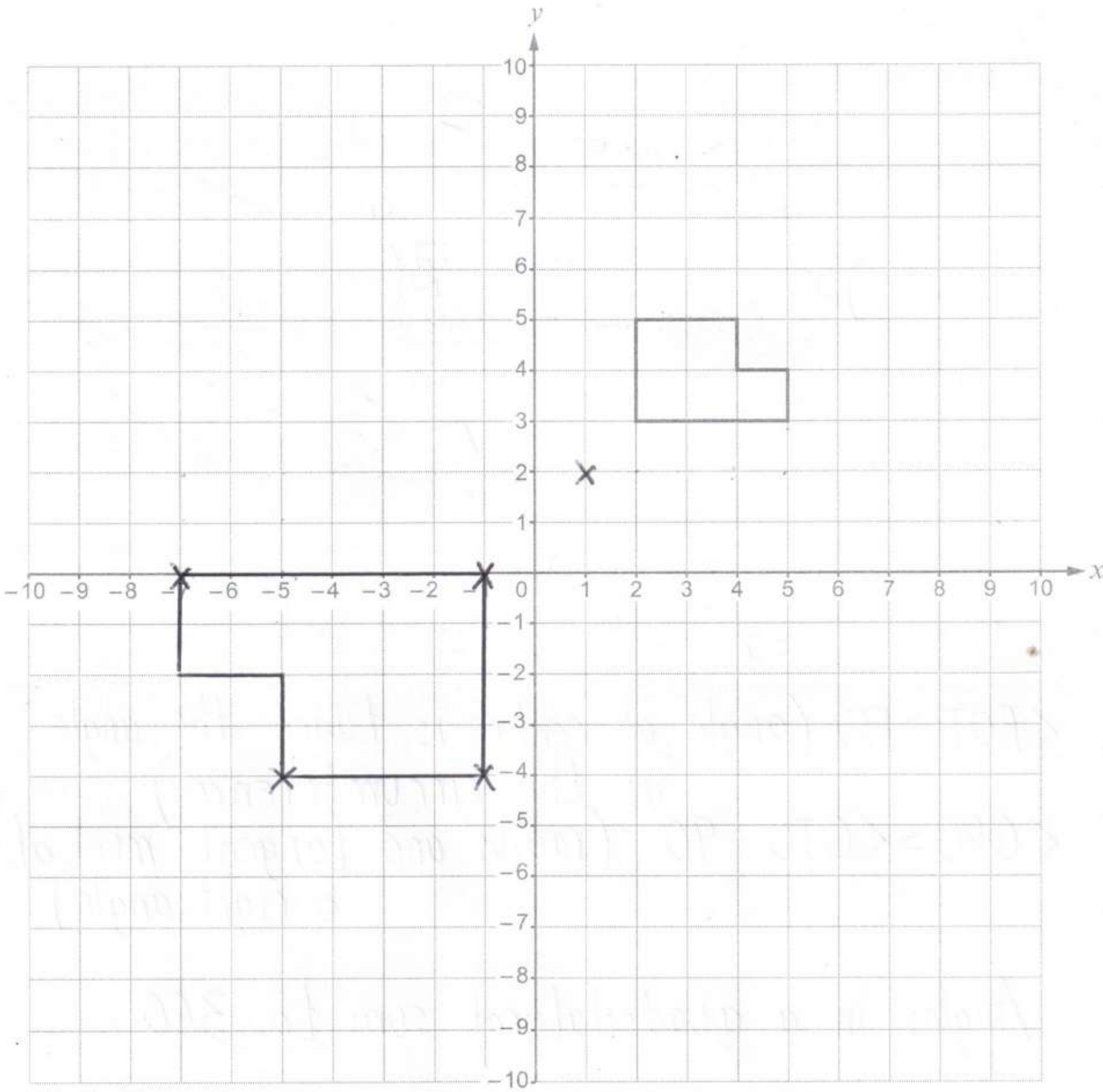
$3000 < \begin{matrix} 3050 \\ 2950 \end{matrix}$ $\qquad$ $720 < \begin{matrix} 725 \\ 715 \end{matrix}$

$S\uparrow = \dfrac{\uparrow D}{\downarrow S} = \dfrac{3050}{715} = 4{\cdot}2657\ldots$

Kian's greatest possible average speed is 4·27 metres per second.

Examiner only

19. Enlarge the shape shown on the grid below by a scale factor of –2.
Use (1, 2) as the centre of enlargement. [3]



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20. The diagram shows points R, S and T on the circumference of a circle with centre O.
The lines QR and QT are both tangents to the circle.

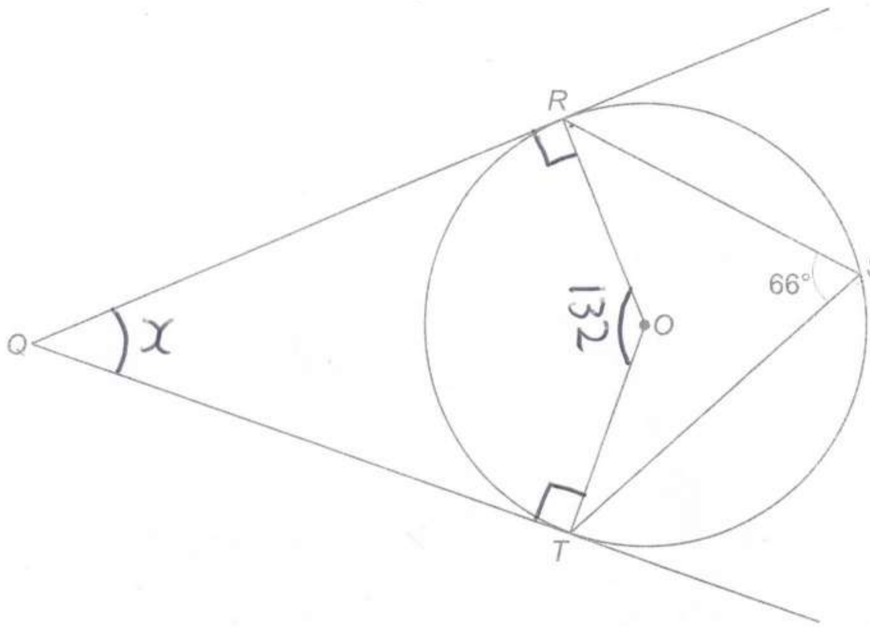


Diagram not drawn to scale

Show that $R\hat{Q}T = 48°$. $= x$
Give a reason for each step of your answer. [4]

$\angle ROT = 132$ (angle at centre is twice the angle at the circumference)

$\angle QRO = \angle QTO = 90$ (radius and tangent meet at a right angle)

Angles in a quadrilateral sum to 360

$x = 360 - 90 - 90 - 132 = 48$

Turn over.

Examiner only

21. Simplify $\dfrac{4x^2-25}{6x^2-15x}$. [4]

$$\frac{(2x+5)\cancel{(2x-5)}}{3x\cancel{(2x-5)}}$$

$$= \frac{2x+5}{3x}$$

22. A line, L, passes through the point (–1, 5) and is perpendicular to $y = \frac{1}{2}x + 5$. $m = \frac{1}{2}$
Find the equation of the line L.
Give your answer in the form $ax + by + c = 0$. [3]

$m_L = -2$ so $y = -2x + c$

$\begin{pmatrix} x = -1 \\ y = 5 \end{pmatrix}$ $5 = -2(-1) + c$

$c = 3$

$y = -2x + 3$

$\Rightarrow 2x + y - 3 = 0$

Examiner only

23.

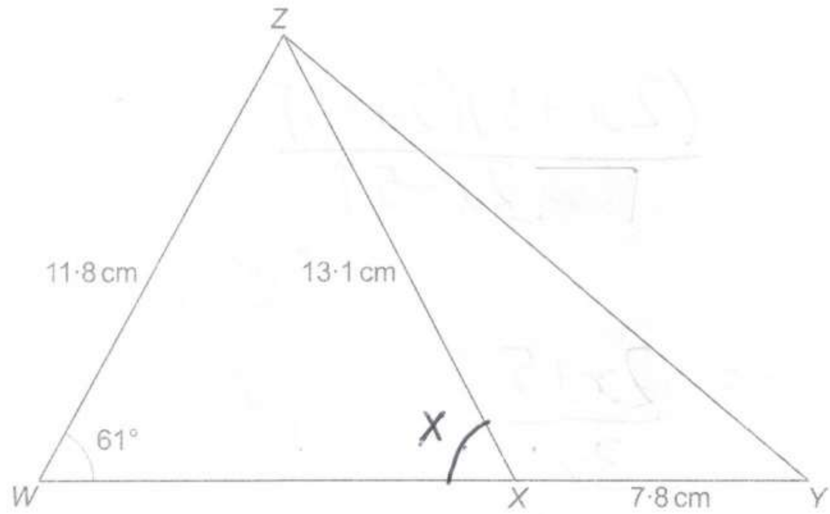


Diagram not drawn to scale

WXY is a straight line.

Calculate the length of *YZ*. [6]

$$\frac{\sin 61}{13 \cdot 1} = \frac{\sin X}{11 \cdot 8} \qquad X = \sin^{-1}\left(\frac{11 \cdot 8 \times \sin 61}{13 \cdot 1}\right)$$

$$= 51 \cdot 98 \ldots$$

$$180 - 51 \cdot 98 = 128 \cdot 02$$

$$YZ^2 = 13 \cdot 1^2 + 7 \cdot 8^2 - 2 \times 13 \cdot 1 \times 7 \cdot 8 \times \cos 128 \cdot 02$$

$$YZ = \sqrt{358 \cdot 32 \ldots}$$

$$= 18 \cdot 93 \text{ cm}$$

Examiner only

24. Hollie makes wooden horses that are mathematically similar.
Two of these horses are shown below.

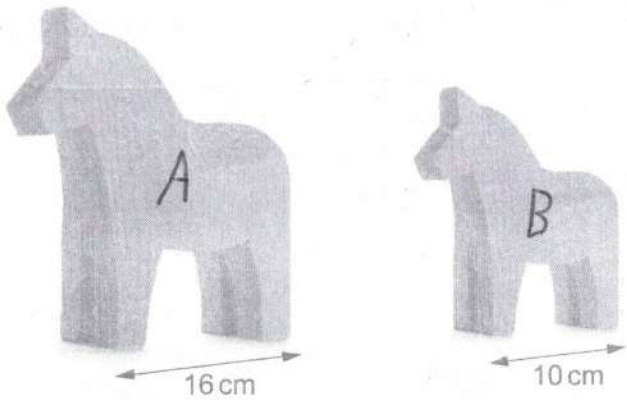


Diagram not drawn to scale

The large horse has a total surface area of $640\,cm^2$.

Hollie uses spray paint to cover the total surface area of each horse.
One can of spray paint covers an area of $9300\,cm^2$ and costs £8.45.

Calculate the cost of buying the **least** number of cans of spray paint required to cover 30 large horses and 50 small horses. [7]

$A \rightarrow B$, linear scale factor $= \frac{10}{16} = \frac{5}{8}$

Surface area of B $= 640 \times \left(\frac{5}{8}\right)^2 = 250$

Total SA $= (30 \times 640) + (50 \times 250)$
$= 31700\ cm^2$

$\frac{31700}{9300} = 3.4\ldots$ so 4 cans needed

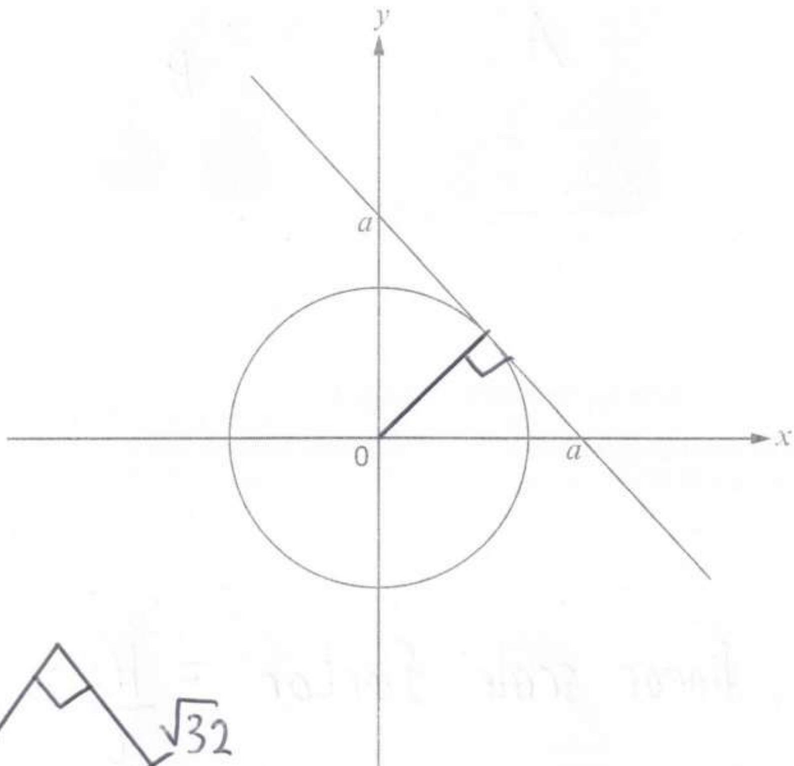
$8.45 \times 4 = £33.80$

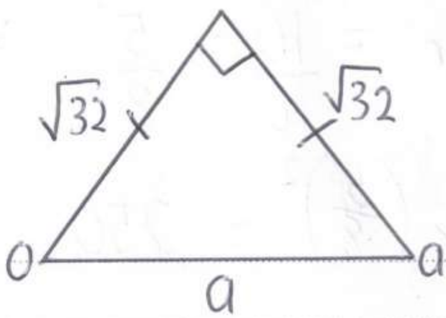
 (C300UB0-1)

25. The diagram shows:

- the circle with equation $x^2 + y^2 = 32$
- a tangent to the circle. The tangent passes through $(a, 0)$ and $(0, a)$.

Calculate the value of a. [3]





$\sqrt{32}^2 + \sqrt{32}^2 = 64$

$a = +\sqrt{64} = \underline{\underline{8}}$

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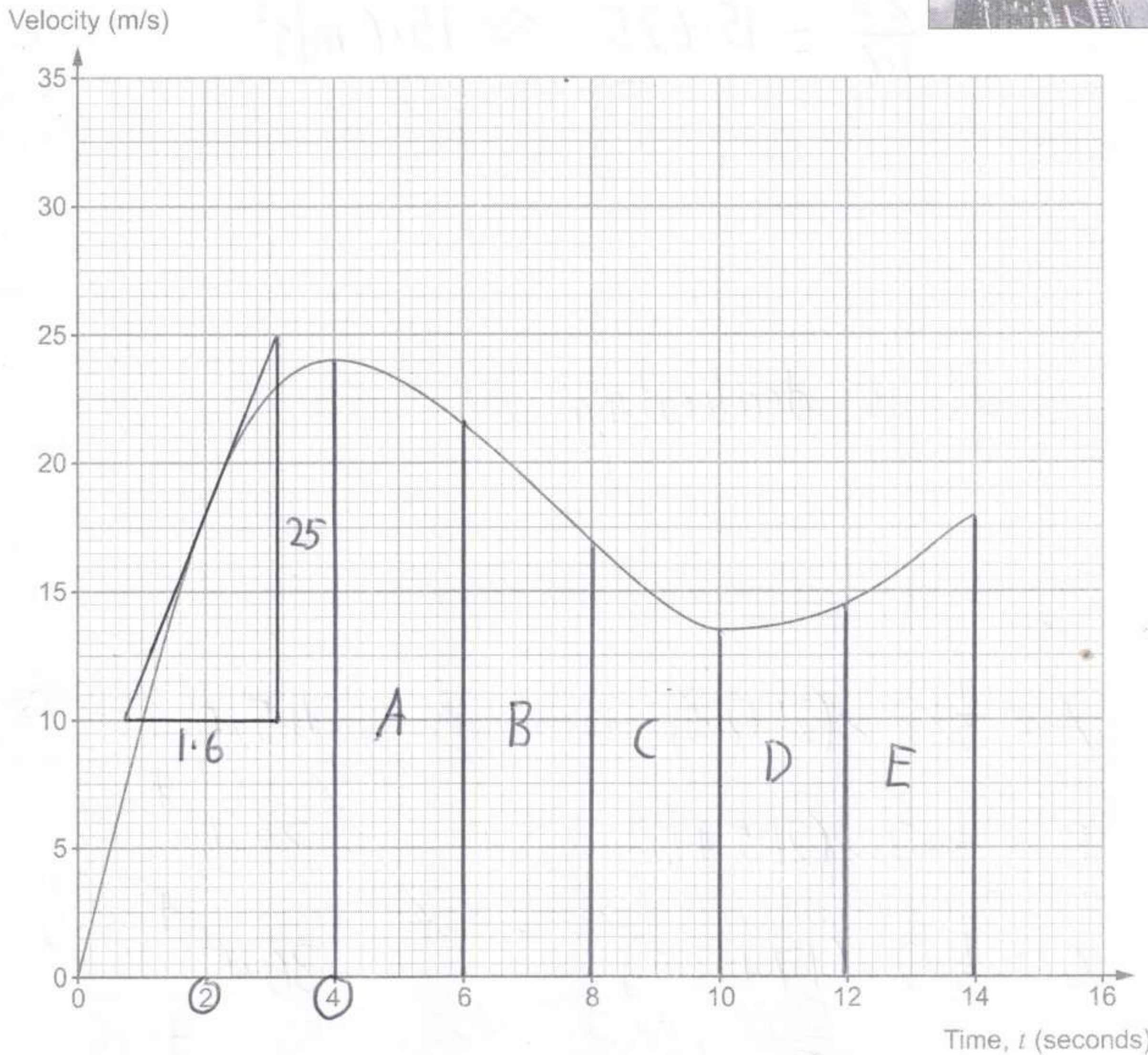
26. One of the attractions at a theme park is a roller coaster.

The graph shows the velocity of the roller coaster, in metres per second, for the first 14 seconds.



(a) Calculate, in **kilometres per hour**, the velocity of the roller coaster at time $t = 4$ seconds. [2]

24m/s $\frac{24 \times 60 \times 60}{1000}$

Velocity is 86·4 kilometres per hour.

 (C300UB0-1)

Examiner only

(b) Calculate an estimate for the acceleration of the roller coaster at time $t = 2$ seconds. Give your answer in m/s^2. [3]

$a = \frac{25}{1.6} = 15.625 \approx 15.6\ m/s^2$

(c) Describe the acceleration of the roller coaster between $t = 4$ and $t = 10$ seconds. [1]

It is decelerating

(d) Use **five** vertical strips of equal width to calculate an estimate for the distance the roller coaster travelled between $t = 4$ and $t = 14$ seconds. Give your answer in metres. [3]

$A = \frac{1}{2} \times 2 \times (24 + 21.5)$ 45.5 +

$B = \frac{1}{2} \times 2 \times (21.5 + 17)$ 38.5 +

$C = \frac{1}{2} \times 2 \times (17 + 13.5)$ 30.5 +

$D = \frac{1}{2} \times 2 \times (13.5 + 14.5)$ 28 +

$E = \frac{1}{2} \times 2 \times (14.5 + 18)$ 32.5

= 175m

Examiner only

27. Find the coordinates of the points of intersection of the curve $3x^2 + 2y^2 = 12$ and the line $y = x - 3$.
Give your solutions correct to 1 decimal place.

You must show all your working. [7]

$3x^2 + 2(x-3)^2 = 12$

$3x^2 + 2(x^2+9-6x) - 12 = 0$

$3x^2 + 2x^2 + 18 - 12x - 12 = 0$

$5x^2 - 12x + 6 = 0$

$x = \dfrac{+12 \pm \sqrt{144 - 120}}{10} = 1{\cdot}689\ldots,\ 0{\cdot}710\ldots$

$x = 1{\cdot}7$
$y = 1{\cdot}7 - 3 = -1{\cdot}3$

$x = 0{\cdot}7$
$y = 0{\cdot}7 - 3 = -2{\cdot}3$

$(1{\cdot}7, -1{\cdot}3)$ and $(0{\cdot}7, -2{\cdot}3)$

END OF PAPER