

Answer ALL TWENTY THREE questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 A tin contains tea bags with a choice of four different flavours of tea.
The four flavours of tea are Assam or Darjeeling or Nilgiri or Rize.

Sara takes at random a tea bag from the tin.

The table shows each of the probabilities that the flavour of the tea Sara takes is Assam or Darjeeling or Rize.

Flavour of tea	Assam	Darjeeling	Nilgiri	Rize
Probability	0.38	0.24		0.16

(a) Work out the probability that the flavour of the tea Sara takes is Nilgiri.

$$1 - 0.78$$

$$= 0.22$$

(2)

(b) Work out the probability that the flavour of the tea Sara takes is either Darjeeling or Rize.

$$0.24 + 0.16$$

$$= 0.4$$

(2)

(Total for Question 1 is 4 marks)



2 Mary saves for a holiday each year.

In 2020 she saved a total of \$720

In 2021, each month she saved \$78

The total amount Mary saved in 2021 was $P\%$ more than the total she saved in 2020

(a) Work out the value of P

$$21 \quad 78 \times 12 = 936$$

$$\text{difference} = 216$$

$$\frac{216}{720} \times 100 = 30$$

30%

(4)



Roberto is going to go on holiday.

He has two coupons that will save him money on his holiday.

Coupon A

18% off the cost of the accommodation

Coupon B

12.5% off the total cost of the accommodation **and** the flights

For Roberto's holiday

the cost of the accommodation is \$1600

the cost of the flights is \$800

Roberto can only use one of the coupons.

He wants to save as much money as he can.

(b) Which of the two coupons, A or B, should he use?

Show your working clearly.

(A) $800 + (1600 \times 0.82) = 2112$

(B) $0.875 \times (1600 + 800) = 2100$

use B, the cost is \$12 cheaper

(3)

(Total for Question 2 is 7 marks)



3 (a) Solve $4y + 5 > 12$

$$4y > 7$$

$$y > \frac{7}{4}$$

(2)

(b) Solve $6x - 5 = \frac{4x - 7}{2}$

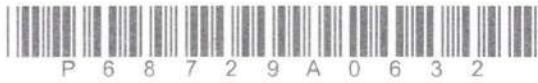
Show clear algebraic working.

$$\begin{aligned} 2(6x - 5) &= 4x - 7 \\ 12x - 10 &= 4x - 7 \\ 8x &= 3 \end{aligned}$$

$$\frac{3}{8}$$

$x = \dots$ (3)

(Total for Question 3 is 5 marks)



4 The diagram shows a regular octagon $ABCDEFGH$ and a regular pentagon $ABIJK$

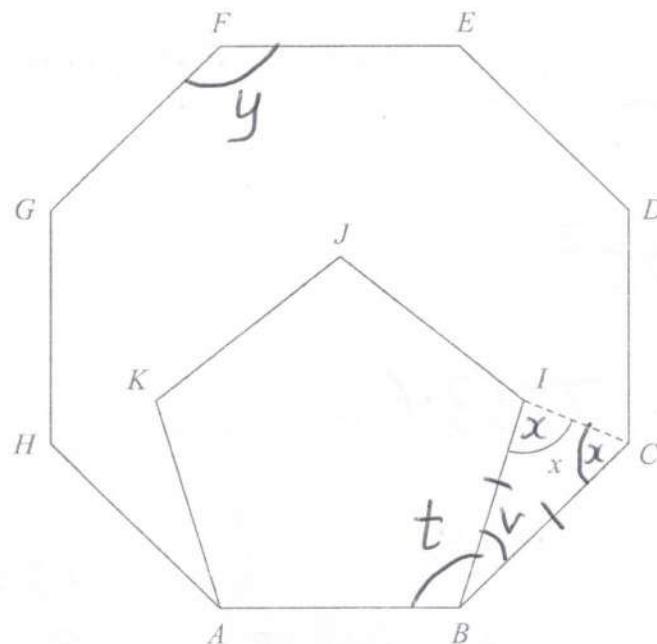


Diagram NOT
accurately drawn

Work out the size of the angle x

$$y = \frac{(8-2) \times 180}{8} = 135$$

$$t = \frac{(5-2) \times 180}{5} = 108$$

$$v = 135 - 108 = 27$$

$$x = \frac{180 - 27}{2}$$

76.5

(Total for Question 4 is 4 marks)

5 Shane invests 7200 dollars for 3 years in a savings account.
He gets 2.5% per year compound interest.

How much money will Shane have in his savings account at the end of 3 years?
Give your answer to the nearest dollar.

$$7200 \times 1.025^3$$
$$= 7753.6\ldots$$

7754 dollars

(Total for Question 5 is 3 marks)



6 (a) Write down the value of x^0

1
(1)

Given that $2^{-3} \times 2^9 = 2^n$

(b) find the value of n

$$-3 + 9 = n$$

6
n =
(1)

Given that $\frac{7^{206} \times 7^m}{7^{214}} = 7^{-3}$

(c) find the value of m

$$7^{206} \times 7^m = 7^{-3} \times 7^{214}$$

$$206 + m = 211$$

5
m =
(2)

(Total for Question 6 is 4 marks)



P 6 8 7 2 9 A 0 9 3 2

7 (a) Write down an equation of the straight line with gradient -3 and which passes through the point with coordinates $(0, 5)$

$$y = -3x + 5$$

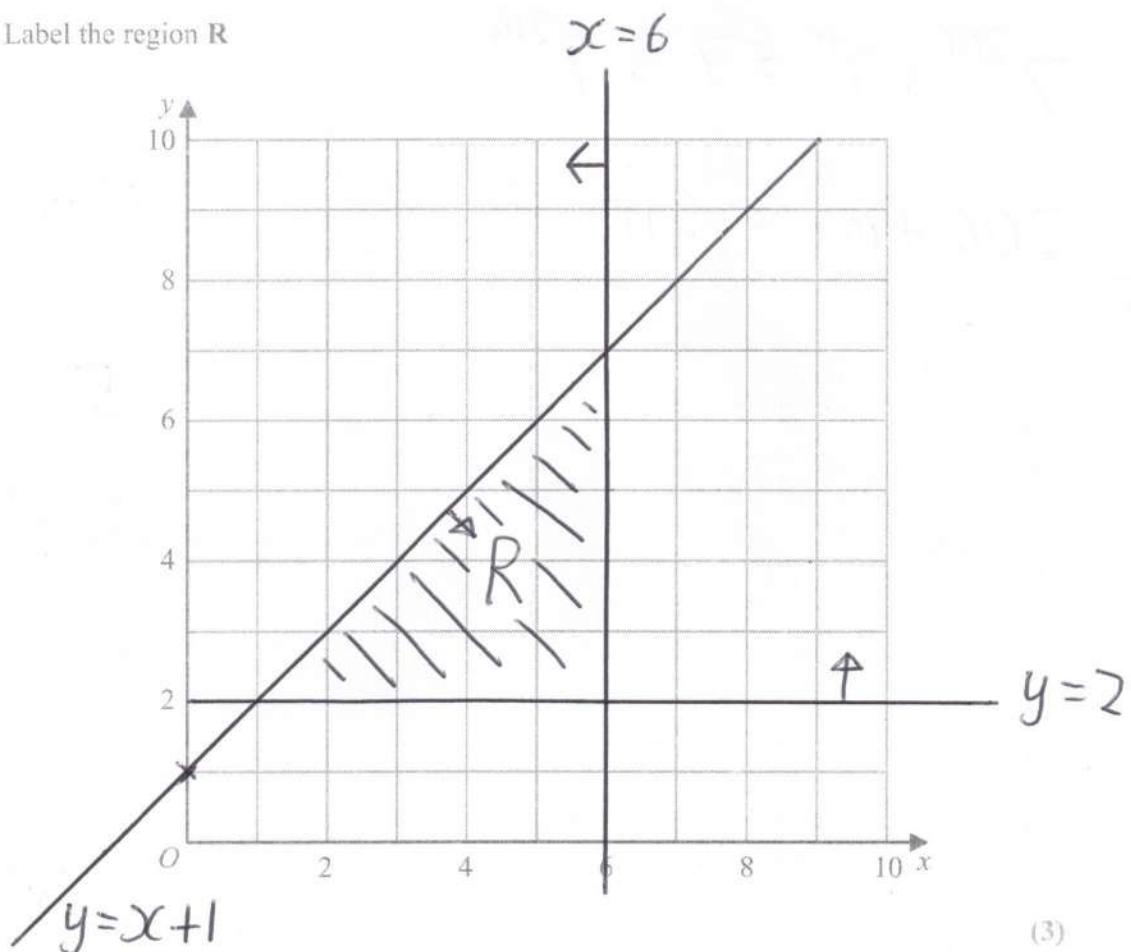
(2)

(b) Show, by shading on the grid, the region defined by **all three** of the inequalities

$$x \leq 6$$

$$y \geq 2$$

$$y \leq x + 1$$

Label the region **R**

(3)

(Total for Question 7 is 5 marks)



8 A scientist is investigating the weight of 50 tigers.

Here is some information about these tigers.

Type of tiger		
	Siberian	Bengal
Number of tigers	22	28
Mean weight of tigers (kg)	260	x

The mean weight of all 50 tigers $(22 \times 260) + 28x = 10900$

Work out the mean weight of the Bengal tigers.

$$(22 \times 260) + 28x = 10900$$

$$28x = 5180$$

$$x = \frac{5180}{28} = 185 \text{ kg}$$

(Total for Question 8 is 3 marks)



P 6 8 7 2 9 A 0 1 1 3 2

9 In the diagram, ABC is a right-angled triangle and DEF is a semicircular arc.

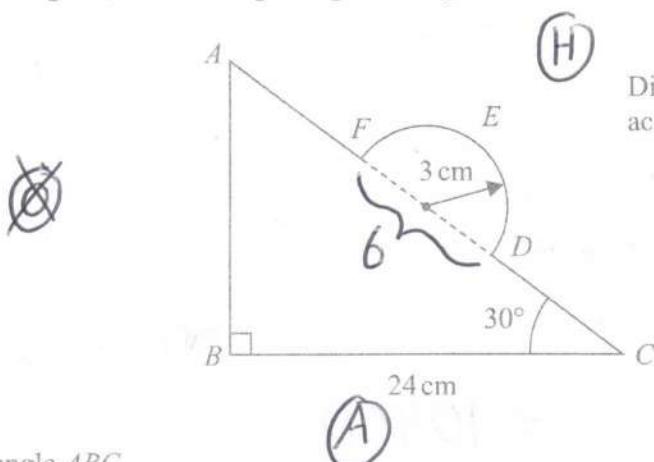


Diagram NOT
accurately drawn

In triangle ABC

$$BC = 24 \text{ cm}$$

$$\text{angle } ABC = 90^\circ$$

$$\text{angle } BCA = 30^\circ$$

The points D and F lie on AC so that DF is the diameter of the semicircular arc DEF .
The radius of the semicircular arc is 3 cm.

Work out the length of $AFEDC$

Give your answer correct to 2 significant figures.

$$AC = \frac{24}{\cos 30} = 16\sqrt{3}$$

$$\text{Circumference of } \cap FED = \frac{2 \times \pi \times 3}{2} = 3\pi$$

$$AFEDC = 16\sqrt{3} - 6 + 3\pi$$

$$\approx 31.1 \text{ cm}$$

$$\underline{\underline{31 \text{ cm}}}$$

5 marks



10 The table gives information about the population and the total amount of money, in dollars, spent on healthcare for two countries in 2016

Country	Total population	Total spent on healthcare (\$)
Austria	8.7×10^6	4.2×10^{10}
Luxembourg	6.3×10^5	3.7×10^9

Work out how much more was spent per person on healthcare in Luxembourg than in Austria.

Give your answer correct to the nearest whole number.

$$L - A$$

$$= \frac{3.7 \times 10^9}{6.3 \times 10^5} - \frac{4.2 \times 10^{10}}{8.7 \times 10^6}$$

$$= 5873.01\dots - 4827.58\dots$$

$$= 1045.4$$

1045

..... dollars

(Total for Question 10 is 3 marks)

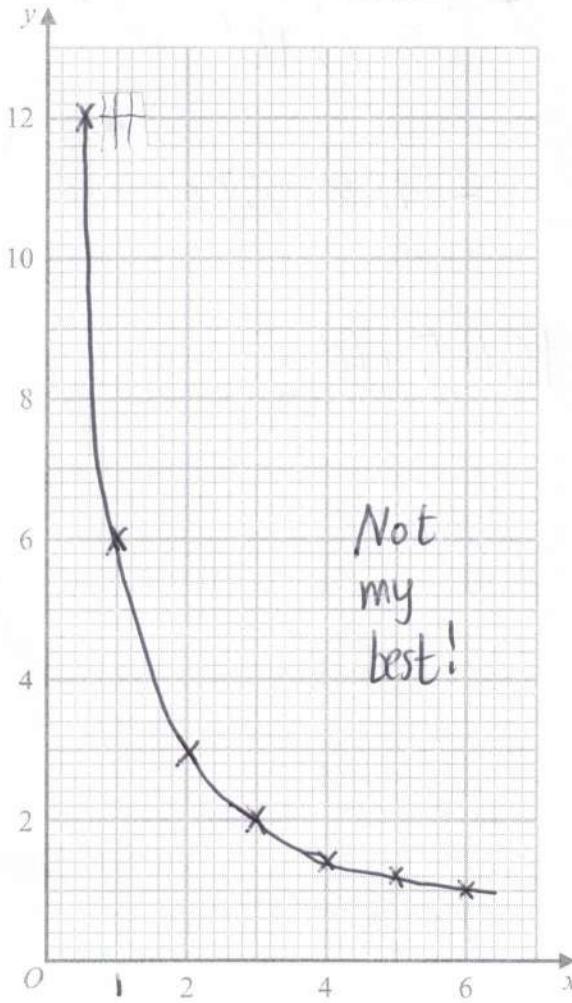


11 (a) Complete the table of values for $y = \frac{6}{x}$

x	0.5	1	2	3	4	5	6
y	12	6	3	2	1.5	1.2	1

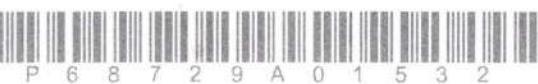
(2)

(b) On the grid, draw the graph of $y = \frac{6}{x}$ for $0.5 \leq x \leq 6$



(2)

(Total for Question 11 is 4 marks)



12 The diagram shows two vertical phone masts, AB and CD , on horizontal ground.

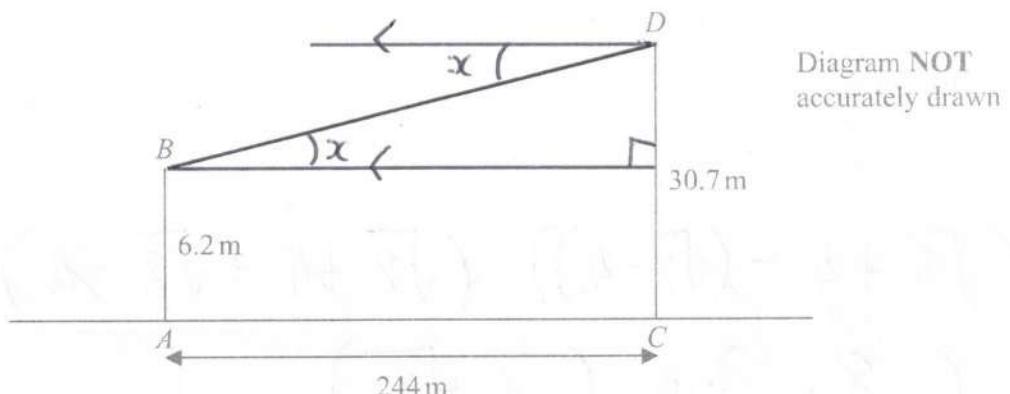


Diagram NOT
accurately drawn

$$AB = 6.2 \text{ m} \quad AC = 244 \text{ m} \quad CD = 30.7 \text{ m}$$

Work out the size of the angle of depression of B from D
Give your answer correct to one decimal place.

$$30.7 - 6.2 = 24.5$$

$$\text{At } A \quad x = \tan^{-1} \left(\frac{24.5}{244} \right)$$

$$= 5.73\ldots$$

5.7

(Total for Question 12 is 3 marks)



$$13 \quad a = \sqrt{8} + 4$$

$$b = \sqrt{8} - 4$$

$(a - b)(a + b)$ can be written in the form $y\sqrt{4y}$

Find the value of y

Show your working clearly.

$$\text{II} \quad (\sqrt{8} + 4 - (\sqrt{8} - 4)) \quad (\sqrt{8} + 4 + \sqrt{8} - 4)$$

$$\text{II} \quad (8) \times (2\sqrt{8})$$

$$\text{II} \quad 16\sqrt{8}$$

$$\text{II} \quad 8\sqrt{4}\sqrt{8}$$

$$\text{II} \quad 8\sqrt{4 \times 8}$$

8

$y =$ _____

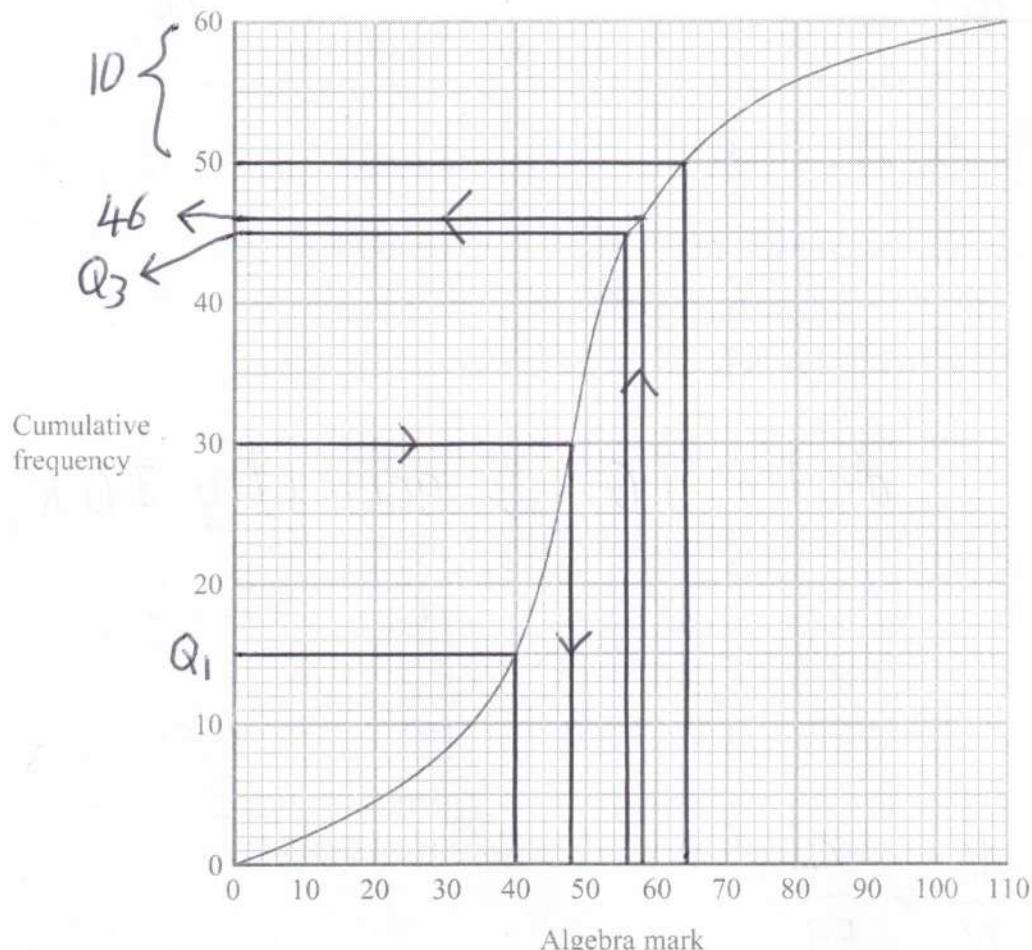
(Total for Question 13 is 3 marks)



P 6 8 7 2 9 A 0 1 7 3 2

14 A group of 60 students each sat an algebra test and a geometry test. Each test was marked out of 110

The cumulative frequency graph gives information about the marks gained by the 60 students in the algebra test.



(a) Use the graph to find an estimate for the median mark in the algebra test.

$[ms: 47-49]$

48

(1)

(b) Use the graph to find an estimate for the number of students who gained 58 marks or less in the algebra test.

46

(1)



(c) Use the graph to find an estimate for the interquartile range of the marks gained in the algebra test.

$$56 - 40$$

[ms: 16-18]

16

(2)

The interquartile range of the marks gained in the geometry test is 9

Luis says

"The students' marks are more spread out in the algebra test than in the geometry test."

(d) Is Luis correct?

Give a reason for your answer.

Yes

algebra IQR > geometry IQR

(1)

To be awarded a grade A in the algebra test, a student had to gain a mark greater than 64

Two students are to be selected at random from the 60 students in the group.

(e) Use the graph to find an estimate for the probability that both of these students were awarded a grade A in the algebra test.

$$\frac{10}{60} \times \frac{9}{59}$$

$$\frac{3}{118}$$

(3)

(Total for Question 14 is 8 marks)



P 6 8 7 2 9 A 0 1 9 3 2

15 Make t the subject of $n^2 = \frac{4d+t^3}{t^3}$

$$n^2 t^3 = 4d + t^3$$

$$n^2 t^3 - t^3 = 4d$$

$$t^3 (n^2 - 1) = 4d$$

$$t^3 = \frac{4d}{n^2 - 1}$$

$$t = \sqrt[3]{\frac{4d}{n^2 - 1}}$$

(Total for Question 15 is 4 marks)



16 The diagram shows quadrilateral $ABCD$

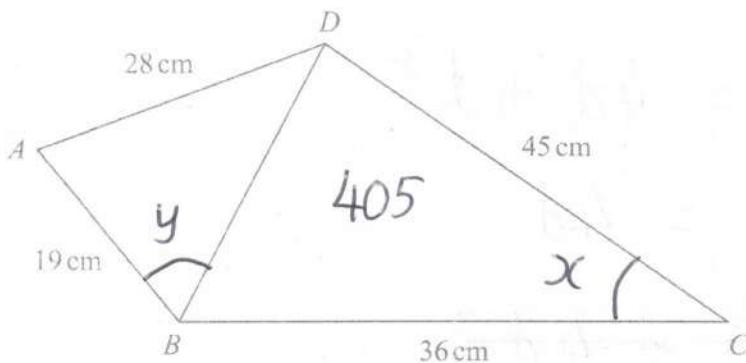


Diagram NOT
accurately drawn

The angle BCD is acute.

Given that the area of triangle $BCD = 405 \text{ cm}^2$

work out the size of angle ABD

Give your answer correct to one decimal place.

$$405 = \frac{1}{2} \times 36 \times 45 \times \sin x$$

$$x = \sin^{-1}(0.5) = 30^\circ$$

$$BD^2 = 36^2 + 45^2 - 2 \times 36 \times 45 \times \cos 30$$

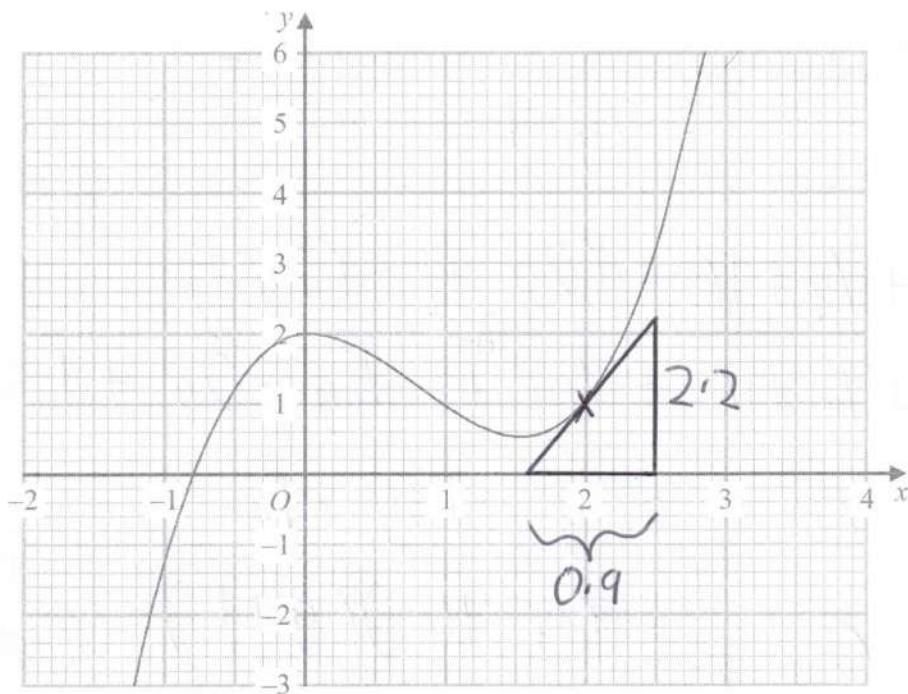
$$BD = \sqrt{515.077\ldots} = 22.695\ldots$$

$$\cos y = \frac{19^2 + 22.695^2 - 28^2}{2 \times 19 \times 22.695}$$

$$y = \cos^{-1}(0.10675\ldots) = 83.87\ldots \\ = 83.9$$

(Total for Question 16 is 5 marks)

17 Part of the curve with equation $y = f(x)$ is shown on the grid.



Find an estimate for the gradient of the curve at the point where $x = 2$
Show your working clearly.

$$\frac{2.2}{0.9} = 2.4$$

[ms: 1.5 → 3]

= 2.4

(Total for Question 17 is 3 marks)



18 The line with equation $2y = x + 1$ intersects the curve with equation $3y^2 + 7y + 16 = x^2 - x$ at the points A and B

Find the coordinates of A and the coordinates of B
Show clear algebraic working.

$$x = 2y - 1$$

$$3y^2 + 7y + 16 = (2y - 1)^2 - (2y - 1)$$

$$3y^2 + 7y + 16 = 4y^2 + 1 - 4y - 2y + 1$$

$$0 = y^2 - 13y - 14$$

$$0 = (y - 14)(y + 1)$$

$$y = 14$$

$$x = (2 \times 14) - 1 \\ = 27$$

$$y = -1$$

$$x = (2 \times -1) - 1 \\ = -3$$

$$(\underline{\hspace{1cm} 27 \hspace{1cm}}, \underline{\hspace{1cm} 14 \hspace{1cm}}) \text{ and } (\underline{\hspace{1cm} -3 \hspace{1cm}}, \underline{\hspace{1cm} -1 \hspace{1cm}})$$

(Total for Question 18 is 5 marks)



19 $ABCD$ is a horizontal rectangular field.

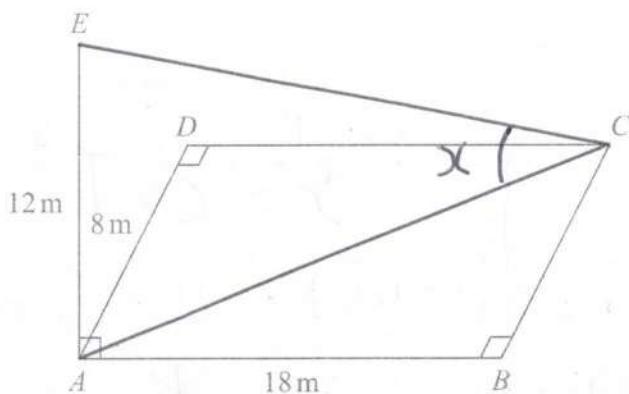
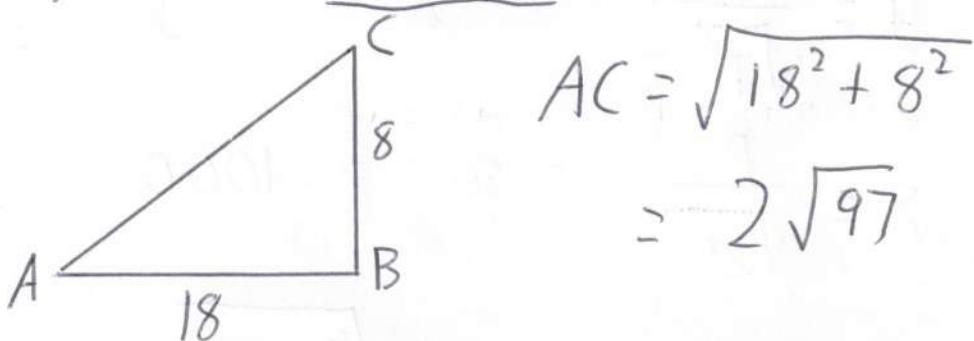


Diagram NOT
accurately drawn

A vertical pole, AE , is placed at the corner A of the field.

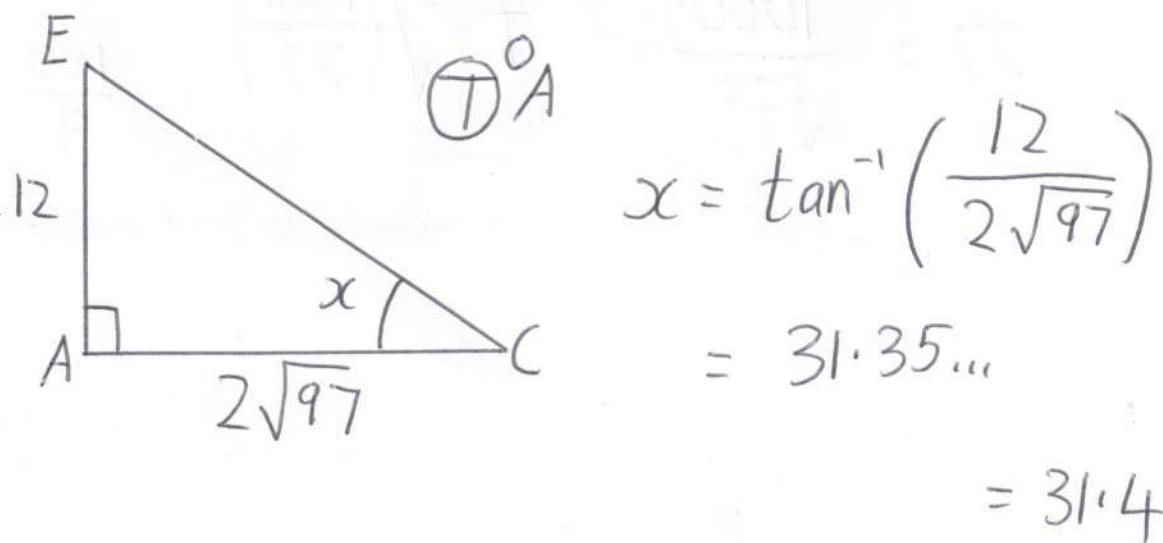
$$AE = 12 \text{ m} \quad AB = 18 \text{ m} \quad AD = 8 \text{ m}$$

Calculate the size of the angle between EC and the plane $ABCD$
Give your answer correct to one decimal place.



$$AC = \sqrt{18^2 + 8^2}$$

$$= 2\sqrt{97}$$



$$x = \tan^{-1} \left(\frac{12}{2\sqrt{97}} \right)$$

$$= 31.35\ldots$$

$$= 31.4$$

(Total for Question 19 is 3 marks)

20 y is inversely proportional to \sqrt{x}
 x is directly proportional to T^3

Given that $y = 8$ when $T = 25$

find the exact value of T when $y = 27$

$$y = \frac{K}{\sqrt{x}}$$

$$x = C \times T^3$$

$$\text{so } y = \frac{K}{\sqrt{C \times T^3}} = \frac{K}{\sqrt{C} \times \sqrt{T^3}}$$

$$y = \frac{P}{\sqrt{T^3}}$$

$$8 = \frac{P}{\sqrt{25^3}}$$

$$\text{so } P = 1000$$

$$27 = \frac{1000}{\sqrt{T^3}}$$

$$T = \sqrt[3]{\left(\frac{1000}{27}\right)^2} = \frac{100}{9}$$

(Total for Question 20 is 4 marks)



21 The diagram shows a solid made from a cylinder and a hemisphere. The cylinder and the hemisphere are both made from the same metal. ✓

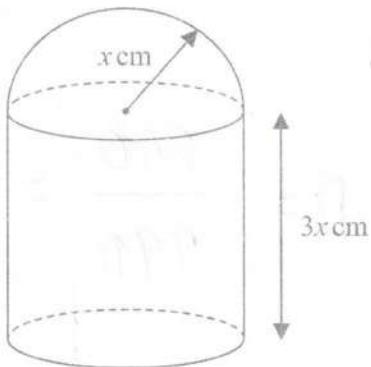


Diagram NOT accurately drawn

The plane face of the hemisphere coincides with the upper plane face of the cylinder. ✓

The radius of the cylinder and the radius of the hemisphere are both x cm. ✓
The height of the cylinder is $3x$ cm. ✓

The total surface area of the solid is $81\pi\text{cm}^2$

The mass of the solid is 840 grams.

The following table gives the density of each of four metals.

Metal	Density (g/cm ³)
Aluminium	2.7
Nickel	8.9
Gold	19.3
Silver	10.5

The metal used to make the solid is one of the metals in the table.

Determine the metal used to make the solid.

Show your working clearly.

$$\begin{aligned}
 SA &= 81\pi = \left(\frac{1}{2} \times 4\pi x^2\right) + \pi x^2 + 2\pi x(3x) \\
 81 &= x^2(2 + 1 + 6) \\
 x^2 &= \frac{81}{9} \quad x = 3
 \end{aligned}$$



$$\text{Vol} = \left(\frac{1}{2} \times \frac{4}{3} \times \pi \times 3^3 \right) + (\pi \times 3^2 \times 9)$$
$$= 99\pi$$

\textcircled{D}^M V

$$D = \frac{840}{99\pi} = 2.70\dots$$

Aluminium

(Total for Question 21 is 6 marks)

Turn over for Question 22



P 6 8 7 2 9 A 0 2 7 3 2

22 ABC is a triangle in which angle $ABC = 90^\circ$

p and q are integers such that

the coordinates of A are $(p, 10)$

the coordinates of B are $(-1, -5)$

the coordinates of C are $(8, q)$

Given that the gradient of AC is $-\frac{6}{7}$

work out the value of p and the value of q

$$m_{AB} = \frac{10 - (-5)}{p - (-1)} = \frac{15}{p + 1}$$

$$m_{BC} = \frac{q - (-5)}{8 - (-1)} = \frac{q + 5}{9}$$

$$\frac{15}{p+1} \times \frac{q+5}{9} = -1$$

$$15(q+5) = -9(p+1)$$

$$15q + 75 = -9p - 9$$

$$15q + 9p = -84$$

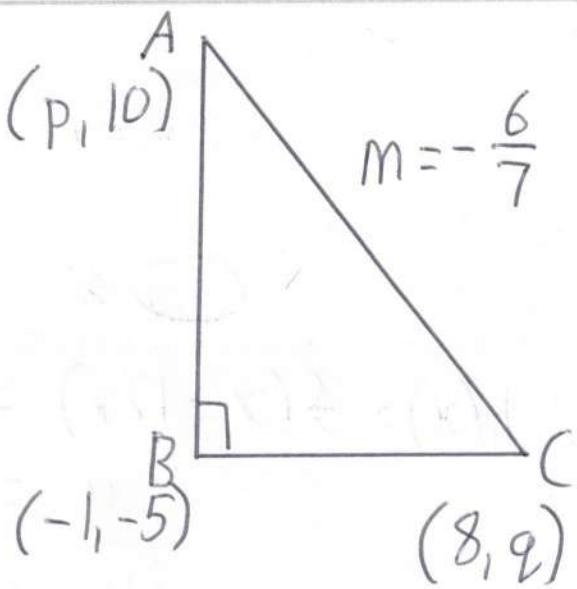
$$5q + 3p = -28 \quad ①$$

$$① \times 2 \quad 10q + 6p = -56$$

$$② \quad 7q - 6p = 22$$

$$\underline{17q = -34}$$

$$q = -2$$



$$m_{AC} = \frac{10 - q}{p - 8} = -\frac{6}{7}$$

$$70 - 7q = -6p + 48$$

$$7q - 6p = 22$$

$$-10 + 3p = -28$$

$$p = -18 \div 3 = -6$$

$$\underline{p = -6, q = -2}$$

23 The functions f and g are such that

$$f(x) = x + 25 \quad g(x) = x^2 - 12x$$

The function h is such that $h(x) = fg(x)$

The domain of h is $\{x \mid x \leq 6\}$ *

Express the inverse function h^{-1} in the form $h^{-1}(x) = \dots$

$$\begin{aligned} h(x) &= f(x^2 - 12x) = x^2 - 12x + 25 \\ y &= (x - 6)^2 - 36 + 25 \\ y + 11 &= (x - 6)^2 \end{aligned}$$

$$\pm\sqrt{y + 11} = x - 6$$

$$x = 6 \pm \sqrt{y + 11}$$

$$* \quad y = 6 - \sqrt{x + 11}$$

$$h^{-1}(x) = 6 - \sqrt{x + 11}$$

(Total for Question 23 is 4 marks)

TOTAL FOR PAPER IS 100 MARKS

