Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel International GCSE

Centre Number | Candidate Number

Monday 7 January 2019

Morning (Time: 2 hours) | Paper Reference **4MA1/1HR**

Mathematics A

Level 1/2
Paper 1HR
Higher Tier

You must have:
Ruler graduated in centimetres and millimetres, protractor, compasses, pen, HB pencil, eraser, calculator. Tracing paper may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – there may be more space than you need.
- **Calculators may be used.**
- You must **NOT** write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over

Pearson

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Answer ALL TWENTY THREE questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 Show that $1\frac{2}{3} + 2\frac{3}{4} = 4\frac{5}{12}$

$= \frac{5}{3} + \frac{11}{4}$

$= \frac{20}{12} + \frac{33}{12}$

$= \frac{53}{12}$ $= 4\frac{5}{12}$

(Total for Question 1 is 3 marks)

2 There are 60 children in a club.

In the club, the ratio of the number of girls to the number of boys is 3 : 1

$\frac{3}{5}$ of the girls play a musical instrument.

$\frac{4}{5}$ of the boys play a musical instrument.

What fraction of the 60 children play a musical instrument?

$60 \times \frac{3}{4} \times \frac{3}{5} = 27$

$60 \times \frac{1}{4} \times \frac{4}{5} = 12$

$27 + 12 = 39$

$\frac{39}{60} = \frac{13}{20}$

(Total for Question 2 is 4 marks)

3

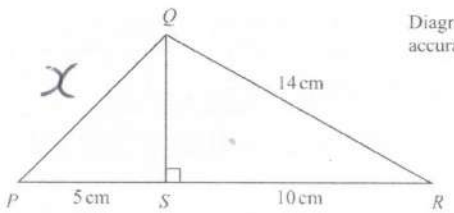

Diagram **NOT** accurately drawn

In triangle PQR,

S is the point on PR such that angle $RSQ = 90°$
$RQ = 14$ cm
$RS = 10$ cm
$SP = 5$ cm

Work out the length of PQ.

$QS = \sqrt{14^2 - 10^2} = \sqrt{96}$

$= 4\sqrt{6}$

$x = \sqrt{(4\sqrt{6})^2 + 5^2}$

$= \sqrt{121}$

$= 11$ cm

(Total for Question 3 is 4 marks)

4 a, a, b and 40 are four numbers.

a is the least number.
40 is the greatest number.

40 − 14 = 26

The range of the four numbers is 14
The median of the four numbers is 30

Work out the value of a and the value of b.

26 26 b 40

30

$a =$ 26

$b =$ 34

(Total for Question 4 is 3 marks)

5 The Shanghai Maglev Train takes 8 minutes to travel a distance of 30.5 kilometres.

Work out the average speed of the train.
Give your answer in kilometres per hour.

$S = \frac{D}{T} = \frac{30.5}{8/60}$

(8 mins = $\frac{8}{60}$ hr)

228.75 kilometres per hour

(Total for Question 5 is 3 marks)

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6 The diagram shows the triangle PQR.

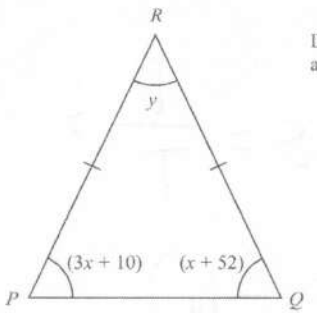

Diagram **NOT** accurately drawn

In the diagram, all the angles are in degrees.

$RP = RQ$

Find the value of y.
Show clear algebraic working.

$3x + 10 = x + 52$

$2x = 42$

$x = 21$

$\angle RPQ = 3 \times 21 + 10 = 73$

$y = 180 - 2 \times 73$

$y =$ 34

(Total for Question 6 is 4 marks)

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7 The diagram shows two water towers in Kuwait.

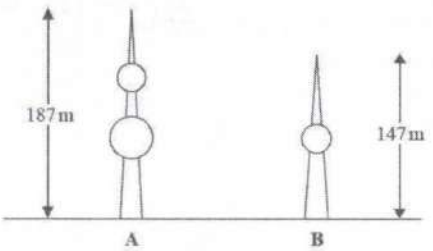

Diagram **NOT** accurately drawn

The real height of tower **A** is 187 m.
The real height of tower **B** is 147 m.

Ahmed makes a scale model of both towers.

The height of tower **A** on the scale model is 90 cm.

Work out the height of tower **B** on the scale model.
Give your answer correct to the nearest centimetre.

$sf = \frac{147}{187}$

$90 \times \frac{147}{187} = 70.748\ldots$

$= 71$

cm

(Total for Question 7 is 3 marks)

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8 Solve the simultaneous equations

$$4x + 2y = 9$$
$$x - 4y = 9$$

(x 2)

+

Show clear algebraic working.

$8x + 4y = 18$

$9x = 27$

$x = \frac{27}{9} = 3$

$3 - 4y = 9$

$-6 = 4y$

$y = \frac{-6}{4} = -\frac{3}{2}$

$x = 3$

$y = -1.5$

(Total for Question 8 is 3 marks)

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9 $N = 480 \times 10^9$

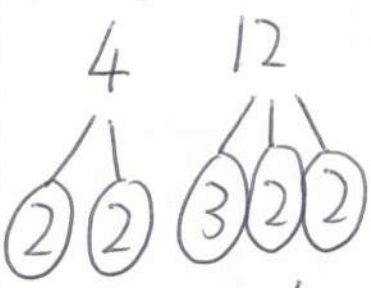
(a) Write N as a number in standard form.

4.8×10^{11}

(1)

(b) Write N as a product of powers of its prime factors.
Show your working clearly.

$48 \times 10^{10} \rightarrow 5^{10} \times 2^{10}$



$= 3 \times 2^4$

$= 5^{10} \times 3 \times 2^{14}$

(3)

(c) Find the largest factor of N that is an odd number.

$5^{10} \times 3$

$= 29\,296\,875$

(1)

(Total for Question 9 is 5 marks)

10 The shape, shown shaded in the diagram, is the region between two semicircles.

$r_1 = 6$

$r_2 = 4$

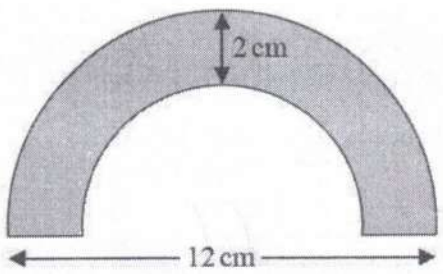


Diagram **NOT** accurately drawn

The diameter of the outer semicircle is 12 cm.
The shape has constant thickness 2 cm.

Calculate the area of the shape.
Give your answer as a multiple of π.

Large semi circle = $\pi \times 6^2 \div 2 = 18\pi$

Small " " = $\pi \times 4^2 \div 2 = 8\pi$

Shaded = $18\pi - 8\pi$

$= 10\pi$

cm^2

(Total for Question 10 is 3 marks)

11 There are 12 boys and 8 girls in a class.
The boys and the girls have some coins.

The mean number of coins that the boys have is 5.5
The girls have a total of 18 coins.

Work out the mean number of coins the 20 children have.

(B) $12 \times 5.5 = 66$

$66 + 18 = 84$

$\frac{84}{20} = \frac{21}{5} = 4.2$

(Total for Question 11 is 3 marks)

12 Here are the first four terms of a sequence of fractions.

$$\frac{1}{1} \quad \frac{2}{3} \quad \frac{3}{5} \quad \frac{4}{7}$$

The numerators of the fractions form the sequence of whole numbers 1 2 3 4 ...
The denominators of the fractions form the sequence of odd numbers 1 3 5 7 ...

(a) Write down an expression, in terms of n, for the nth term of this sequence of fractions.

1 2 3 4 = n

1 3 5 7 = 2n-1

(+2 +2 +2)

$$\frac{n}{2n-1}$$

(2)

(b) Using algebra, prove that when the square of any odd number is divided by 4 the remainder is 1

odd = 2n + 1

$$(2n+1)^2 = 4n^2 + 4n + 1$$

$$= 4(n^2 + n) + 1$$

divisible by 4 — remainder

(3)

(Total for Question 12 is 5 marks)

13 A curve **C** has equation $y = x^3 - x^2 - 8x + 12$

(a) Find $\frac{dy}{dx}$

$\frac{dy}{dx} = 3x^2 - 2x - 8$

(2)

The curve **C** has two turning points.

(b) Work out the x coordinates of the two turning points. Show your working clearly.

$\rightarrow \frac{dy}{dx} = 0$

$3x^2 - 2x - 8 = 0$

$(3x + 4)(x - 2) = 0$

$x = -\frac{4}{3}, \quad x = 2$

(3)

(c) Show that the x-axis is a tangent to the curve **C**.

At $x = 2$, $y = 2^3 - 2^2 - 8 \times 2 + 12$

$= 8 - 4 - 16 + 12$

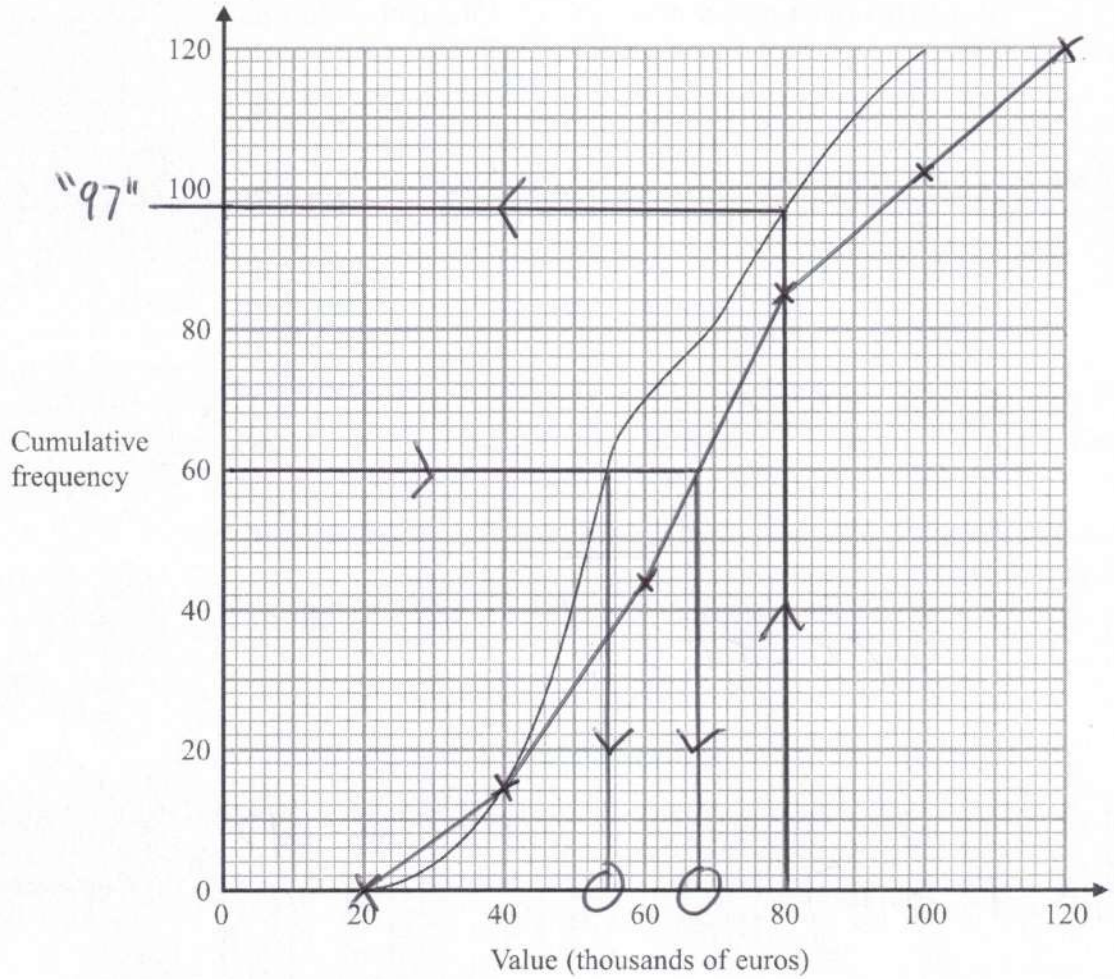
$= 0$

hence (2,0) is a turning point = tangent

(2)

(Total for Question 13 is 7 marks)

14 The cumulative frequency diagram gives information about the values, in thousands of euros, of 120 apartments in 2015



(a) Find an estimate for the number of these apartments with a value of 80 thousand euros or less in 2015

97

(1)

[ms 96 → 98]

The table gives information about the values, in thousands of euros, of the same 120 apartments in 2018

Value in thousands of euros (v)	Cumulative frequency
$0 < v \leqslant 20$	0
$0 < v \leqslant 40$	15
$0 < v \leqslant 60$	44
$0 < v \leqslant 80$	85
$0 < v \leqslant 100$	102
$0 < v \leqslant 120$	120

(b) On the grid opposite, draw a cumulative frequency diagram for this information.

(2)

(c) Find an estimate for the increase in the median value for these apartments from 2015 to 2018

$68 - 55$

$= 13$

[ms: 13 → 15]

thousand euros

(2)

(Total for Question 14 is 5 marks)

15 (a) Simplify $(3x^2y^5)^4$

$3^4 \quad (x^2)^4 \quad (y^5)^4$

$81x^8y^{20}$

(2)

(b) Expand and simplify $4n(n-3)(n+5)$

$4n(n^2 - 3n + 5n - 15)$

$4n(n^2 + 2n - 15)$

$4n^3 + 8n^2 - 60n$

(2)

(c) Factorise $4c^2 - 9d^2$

$(2c + 3d)(2c - 3d)$

(1)

(d) Simplify fully $\dfrac{x^2 - 7x + 12}{4x - x^2}$

$= \dfrac{(x-4)(x-3)}{x(4-x)} = \dfrac{(4-x)(3-x)}{x(4-x)}$

$= \dfrac{3-x}{x}$

(3)

(Total for Question 15 is 8 marks)

16 There are 12 beads in a bag.

7 of the beads are red.
3 of the beads are green.
2 of the beads are yellow.

Lucy takes at random a bead from the bag and keeps it.
Then Julian takes at random a bead from the bag.

(a) Work out the probability that they each take a yellow bead.

$$P(YY) = \frac{2}{12} \times \frac{1}{11} = \frac{2}{132}$$

$$= \frac{1}{66}$$

(2)

(b) Work out the probability that the beads they take are **not** the same colour.

$$\text{Not same} = 1 - \text{same}$$

$$= 1 - \frac{1}{66} - GG - RR$$

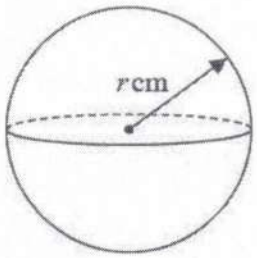
$$= \frac{65}{66} - \left(\frac{3}{12} \times \frac{2}{11}\right) - \left(\frac{7}{12} \times \frac{6}{11}\right)$$

$$= \frac{41}{66}$$

(3)

(Total for Question 16 is 5 marks)

17 Here are a solid sphere and a solid cylinder.



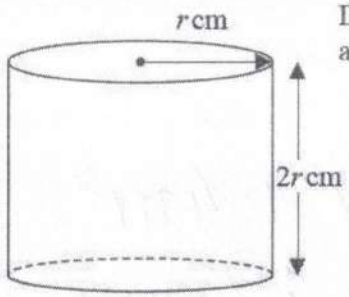


Diagram **NOT** accurately drawn

The radius of the sphere is r cm.
The radius of the cylinder is r cm.
The height of the cylinder is $2r$ cm.

The total surface area of the cylinder is $k\pi$ cm^2

(a) Find an expression for k in terms of r.

$SA = \square + OO$

$= 2r(2\pi r) + 2 \times \pi r^2$

$= 4\pi r^2 + 2\pi r^2$

$K = 6r^2 \pi \text{ cm}^2$

(2)

(b) Show that the ratio

total surface area of the cylinder : total surface area of the sphere

is the same as the ratio

volume of the cylinder : volume of the sphere

SA : $6\pi r^2 : 4\pi r^2$

$3 : 2$

Vol C : Vol S

$\pi r^2(2r) : \frac{4}{3}\pi r^3$

$2\pi r^3 : \frac{4}{3}\pi r^3$ (×3)

$6 : 4$

$3 : 2$

(3)

(Total for Question 17 is 5 marks)

18 Show that $\frac{\sqrt{8}}{\sqrt{8}-2}$ can be written in the form $n + \sqrt{n}$, where n is an integer.

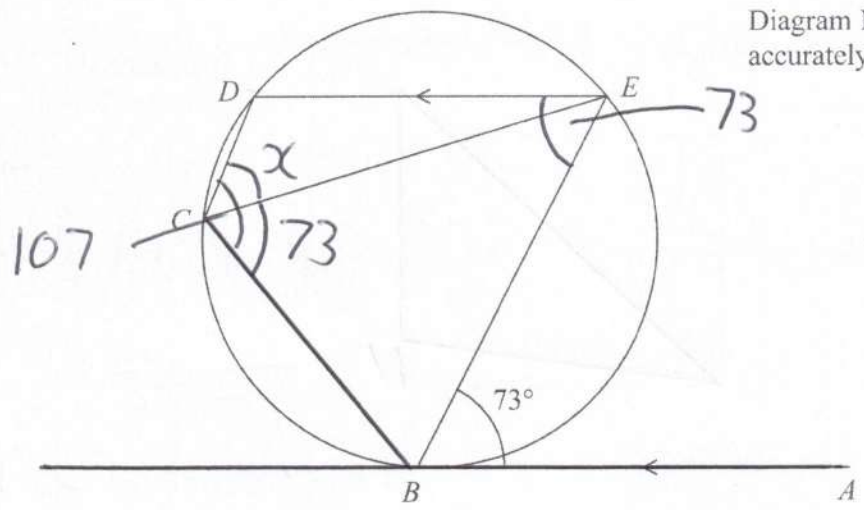
Show your working clearly.

$\frac{\sqrt{8}}{\sqrt{8}-2} \times \frac{\sqrt{8}+2}{\sqrt{8}+2}$

$= \frac{8 + 2\sqrt{8}}{8-4} = \frac{8+4\sqrt{2}}{4} = 2+\sqrt{2}$

(Total for Question 18 is 3 marks)

19



B, *C*, *D* and *E* are points on a circle.

AB is the tangent at *B* to the circle.
AB is parallel to *ED*.
Angle *ABE* = 73°

Work out the size of angle *DCE*.
Give a reason for each stage of your working.

$\angle DEB = 73$ (alternate angles are equal)

$\angle DCB = 180 - 73 = 107$ (opposite angles of a cyclic quadrilateral sum to 180)

$\angle BCE = 73$ (alternate segment theorem)

$\angle DCE = 107 - 73$

$= 34°$

(Total for Question 19 is 5 marks)

20 Here is a cube *ABCDEFGH*.

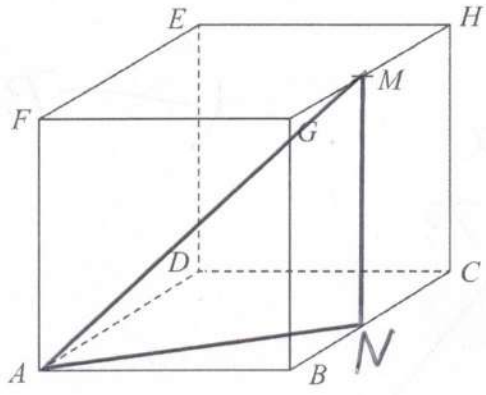
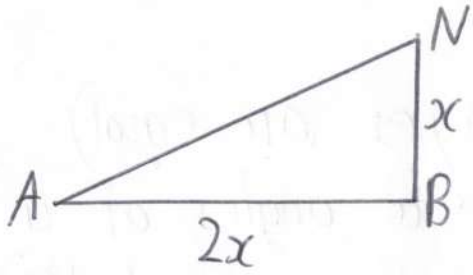


Diagram **NOT** accurately drawn

M is the midpoint of the edge *GH*.

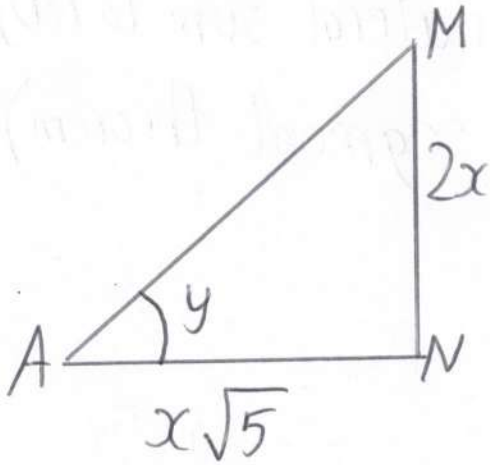
Find the size of the angle between the line *MA* and the plane *ABCD*.
Give your answer correct to 1 decimal place.

Let side of cube = $2x$



$AN = \sqrt{(2x)^2 + x^2}$

$= \sqrt{5x^2} = x\sqrt{5}$



$y = \tan^{-1}\left(\frac{2x}{x\sqrt{5}}\right)$

$= 41.810...$

$= 41.8°$

(Total for Question 20 is 4 marks)

21 Here is a triangle XYZ.

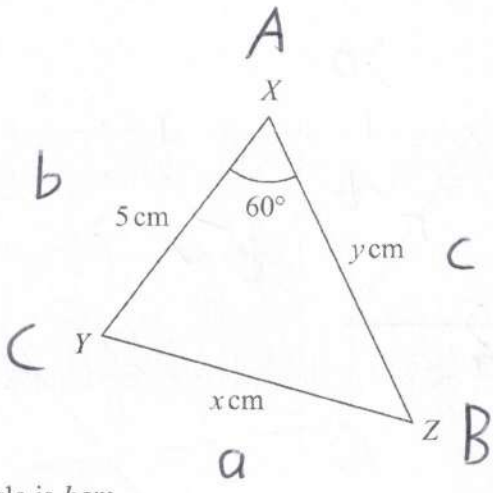


Diagram **NOT** accurately drawn

The perimeter of the triangle is k cm.

Given that $x = y - 1$
find the value of k.
Show your working clearly.

$a^2 = b^2 + c^2 - 2bc\cos A$

$x^2 = 25 + y^2 - 10y \cos 60$

$(y-1)^2 = y^2 + 25 - 10y \times \frac{1}{2}$

$y^2 - 2y + 1 = y^2 + 25 - 5y$

$3y = 24$

$y = 8$

$K = \text{perimeter} = 5 + 8 + 7$

$k =$ 20

(Total for Question 21 is 5 marks)

22 *ABCDEF* is a regular hexagon.

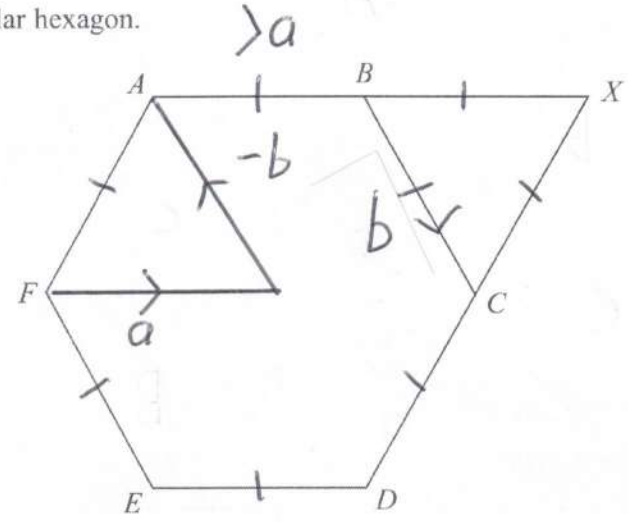


Diagram **NOT** accurately drawn

ABX and *DCX* are straight lines.

$\overrightarrow{AB} = \mathbf{a}$ $\overrightarrow{BC} = \mathbf{b}$

Find $\overrightarrow{EX}$ in terms of **a** and **b**.
Give your answer in its simplest form.

$\overrightarrow{FA} = \overrightarrow{DC} = \overrightarrow{CX} = a - b$

$\overrightarrow{EX} = \overrightarrow{ED} + \overrightarrow{DC} + \overrightarrow{CX}$

$= a + a - b + a - b$

$= 3a - 2b$

(Total for Question 22 is 4 marks)

23 The function f is defined as $f(x) = \dfrac{\sqrt{x^2 + k^2}}{x}$ for $x > 0$ and where k is a positive number.

(a) Find the value of p for which $f^{-1}(p) = k$

$$y = \frac{\sqrt{x^2+k^2}}{x}$$

$$y^2x^2 = x^2 + k^2$$

$$y^2x^2 - x^2 = k^2$$

$$x^2(y^2-1) = k^2$$

$$x^2 = \frac{k^2}{y^2-1}$$

$$f^{-1}(x) = \frac{k}{\sqrt{x^2-1}}$$

$$f^{-1}(p) = \frac{k}{\sqrt{p^2-1}} = k$$

$$\text{so } \sqrt{p^2-1} = 1$$

$$p^2 = 2$$

$p =$ $\sqrt{2}$ **(3)**

The function g is defined as $g(x) = x^2$ for $x > 0$

(b) Given that $gf(a) = k$ for $k > 1$
find an expression for a in terms of k.

$$gf(a) = g\left(\frac{\sqrt{a^2+k^2}}{a}\right) = \left(\frac{\sqrt{a^2+k^2}}{a}\right)^2$$

$$k = \frac{a^2+k^2}{a^2}$$

$$a^2k - a^2 = k^2$$

$$a^2(k-1) = k^2$$

$$a^2 = \frac{k^2}{k-1}$$

$a =$ $\dfrac{k}{\sqrt{k-1}}$ **(3)**

(Total for Question 23 is 6 marks)

TOTAL FOR PAPER IS 100 MARKS