

Answer ALL TWENTY TWO questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1  $w = 5y^2 - y^3$

(a) Work out the value of  $w$  when  $y = -2$ 

$$\begin{aligned}
 &= 5(-2)^2 - (-2)^3 \\
 &= 20 + 8
 \end{aligned}$$

$$w = \dots \quad (2)$$

(b) Factorise fully  $8p^2 - 2p$

$$2p(4p - 1) \quad (2)$$

(c) Expand  $4t(3t - 2)$

$$12t^2 - 8t \quad (2)$$

(d) Expand and simplify  $(5x - 2)(x + 4)$

$$5x^2 + 20x - 2x - 8$$

$$5x^2 + 18x - 8 \quad (2)$$

(Total for Question 1 is 8 marks)



2 The diagram shows a rectangle  $ABCD$  and a semicircle with diameter  $AB$  where  $AB = 12\text{ cm}$ . The point  $E$  lies on  $DC$  and also on the semicircle.

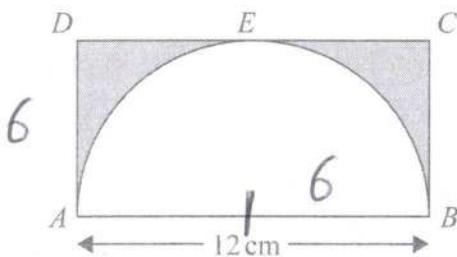


Diagram **NOT**  
accurately drawn

Work out the area of the shaded region.  
Give your answer correct to 3 significant figures.

$$\begin{aligned}A &= \square - \Delta \\&= (12 \times 6) - (\pi \times 6^2 \div 2) \\&= 15.45\ldots\end{aligned}$$

15.5  
cm<sup>2</sup>

(Total for Question 2 is 3 marks)



3  $\mathcal{E} = \{21, 22, 23, 24, 25, 26, 27, 28, 29, 30\}$

$A = \{22, 24, 26, 28, 30\}$

$B = \{21, 24, 27, 30\}$

(a) List the members of the set

(i)  $A \cap B$

24, 30

(ii)  $A'$

21, 23, 25, 27, 29

(2)

$C = \{23, 25, 29\}$

(b) Using set notation, find an expression for  $C$  in terms of  $A$  and  $B$ .

$(A \cup B)'$  or  $A' \cap B'$

(1)

(Total for Question 3 is 3 marks)

4 (a) Simplify  $(3k^2)^4$

$3^4 = 81$   $\sqrt{2 \times 4} = 8$

$81k^8$

(2)

(b) Simplify  $(21m^4n) \div (3n^5)$

$\frac{21}{3} = 7$   $\sqrt{1 - - 5} = 6$

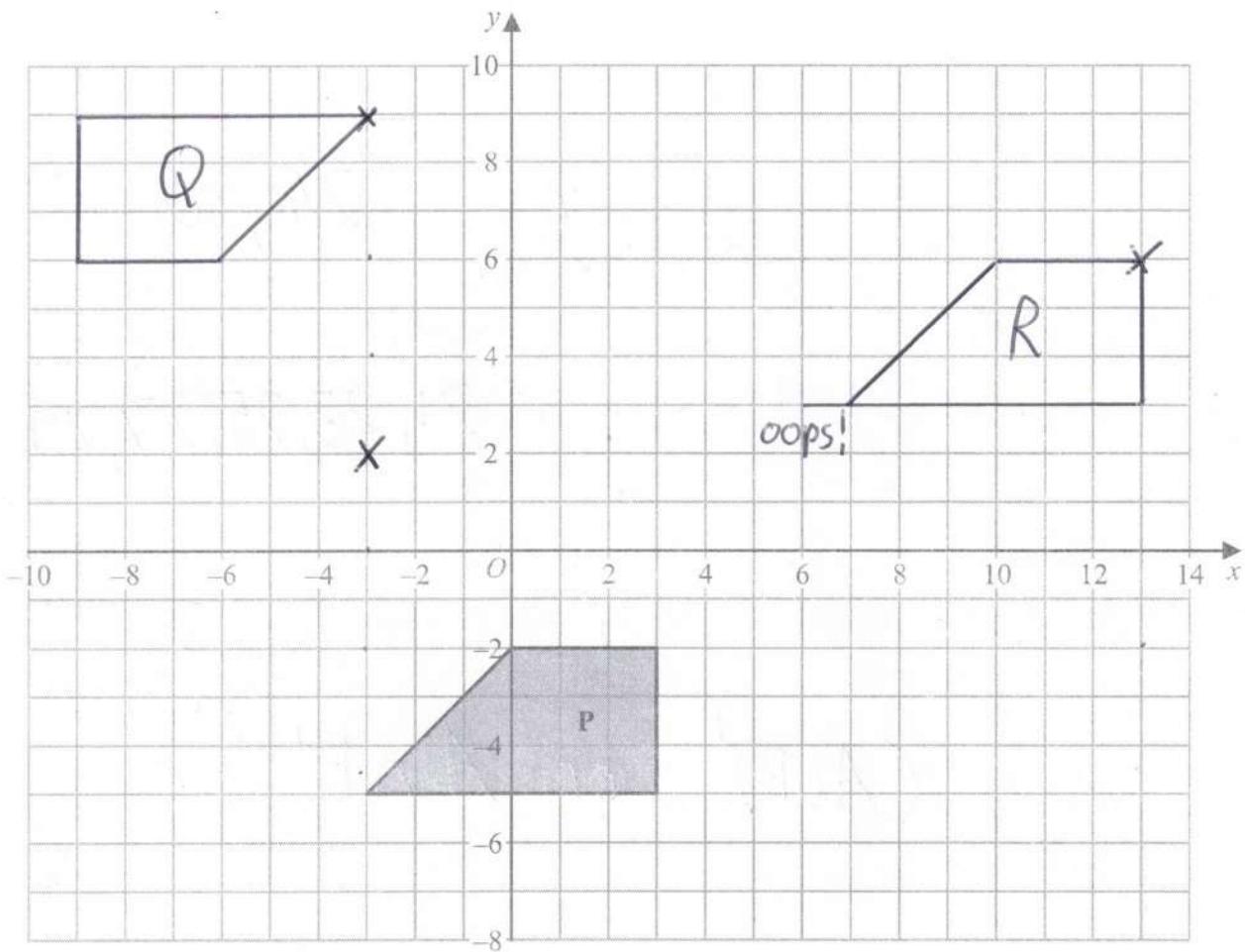
$7m^4n^6$

(2)

(Total for Question 4 is 4 marks)



5 Here is a shape **P** drawn on a grid of squares.



(a) On the grid, rotate shape **P**  $180^\circ$  about the point  $(-3, 2)$   
Label the new shape **Q**.

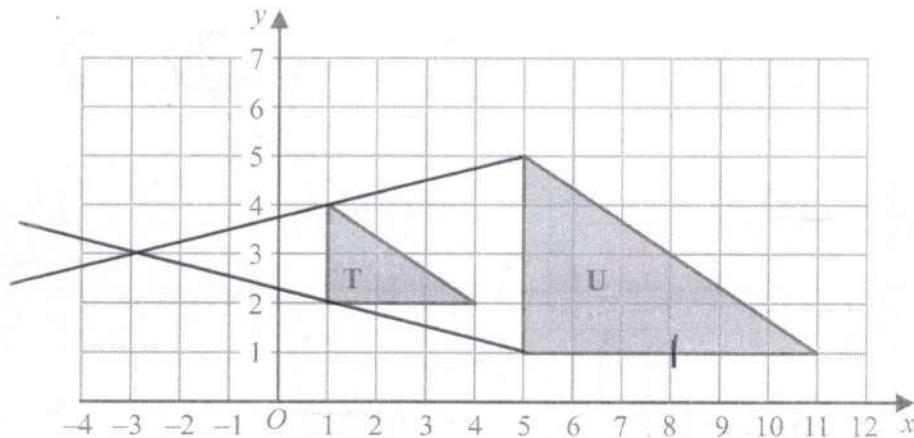
(2)

(b) On the grid, translate shape **P** by the vector  $\begin{pmatrix} 10 \\ 8 \end{pmatrix}$   $\rightarrow 10$   
Label the new shape **R**.

(1)



Here are triangle T and triangle U drawn on a grid of squares.



(c) Describe fully the single transformation that maps triangle T onto triangle U.

Enlargement, scale factor = 2, centre  $(-3, 3)$

(Total for Question 5 is 6 marks)

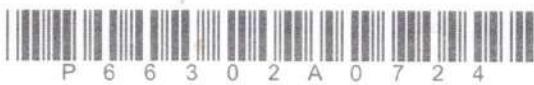
6 On Wednesday, the price of 1 litre of petrol was £1.26  
 The price of petrol on Wednesday was 5% more than the price of petrol on the previous Monday.

Calculate the price of 30 litres of petrol on the previous Monday.

$$30 \times 1.26 \div 1.05$$

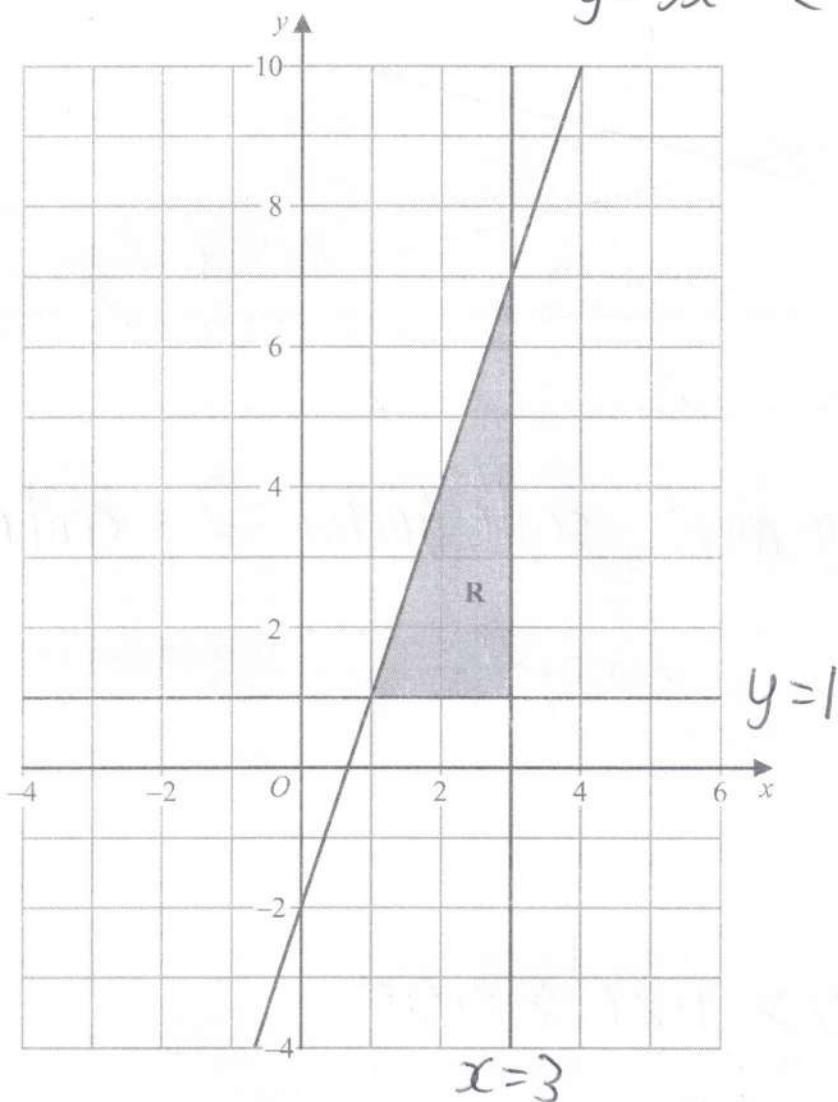
36  
£

(Total for Question 6 is 3 marks)



7 The shaded region **R**, shown in the diagram below, is bounded by the straight line with equation  $y = 3x - 2$  and by two other straight lines.

Write down the three inequalities that define region **R**.

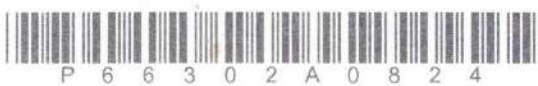


$$x \leq 3$$

$$y \geq 1$$

$$y \leq 3x - 2$$

(Total for Question 7 is 3 marks)



8 The table gives the length of the coastline, in kilometres, of each of five oceans.

Ocean	Length of coastline (km)
Arctic	$4.539 \times 10^4$
Atlantic	$1.119 \times 10^5$
Pacific	$1.357 \times 10^5$
Indian	$6.653 \times 10^4$
Southern	$1.797 \times 10^4$

(a) Which ocean has the greatest length of coastline?

Pacific

(1)

(b) Calculate the difference between the length of the Atlantic Ocean's coastline and the length of the Southern Ocean's coastline.

Give your answer in standard form.

$$1.119 \times 10^5 - 1.797 \times 10^4$$

$$= 93930$$

$$9.393 \times 10^4$$

km

(2)

(Total for Question 8 is 3 marks)

9 Solve  $x^2 - 21x + 20 = 0$

Show your working clearly.

$$(x-1)(x-20) = 0$$

$$x = 1, 20$$

(Total for Question 9 is 3 marks)



10 A mathematics teacher at a school asked a group of students how far, in kilometres, each student had travelled to get to school that day.

The table gives information about their answers.

Distance travelled (d km)	Number of students
$0 < d \leq 2$	1
$2 < d \leq 4$	3
$4 < d \leq 6$	5
$6 < d \leq 8$	7
$8 < d \leq 10$	9

$$\begin{array}{r}
 x \\
 33 \\
 40 \\
 42 \\
 45 \\
 \hline
 = x +
 \end{array}
 \left. \begin{array}{l}
 x \\
 x + 160
 \end{array} \right\}$$

The teacher calculated that an estimate for the mean distance travelled by the whole group of students was 4.25 km.

Work out the value of  $x$ .

Show your working clearly.

$$\frac{x + 160}{x + 30} = 4.25$$

$$x + 160 = 4.25x + 127.5$$

$$32.5 = 3.25x$$

$$x = \frac{32.5}{3.25}$$

$$10$$

$$x = \dots$$

(Total for Question 10 is 4 marks)



11 A circle centre  $O$  has radius 9 cm.

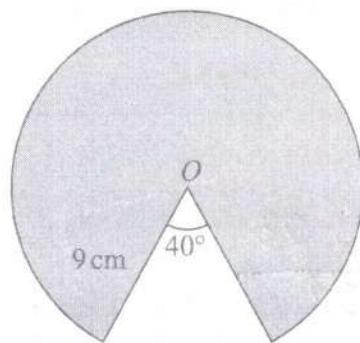


Diagram NOT  
accurately drawn

Calculate the perimeter of the shaded sector of the circle.  
Give your answer correct to 3 significant figures.

$$\left(2\pi \times 9 \times \frac{320}{360}\right) + 9 + 9$$

$$= 68.26...$$

68.3

cm

(Total for Question 11 is 4 marks)

12 Solve the simultaneous equations  $2x + 7y = 17$   
 $5x + 3y = -1$

-①  
-②

Show clear algebraic working.

$$\begin{array}{r} \textcircled{1} \times 5 \quad 10x + 35y = 85 \\ \textcircled{2} \times 2 \quad 10x + 6y = -2 \\ \hline 29y = 87 \end{array}$$

$$y = \frac{87}{29} = 3 \quad 5x + 9 = -1$$

$$5x = -10 \quad x = -2$$

$$x = -\frac{10}{5} \quad y = 3$$

(Total for Question 12 is 4 marks)



13 The diagram shows two hot air balloons.

$A$  is a point on the base of one of the balloons and  $B$  is a point on the base of the other balloon.

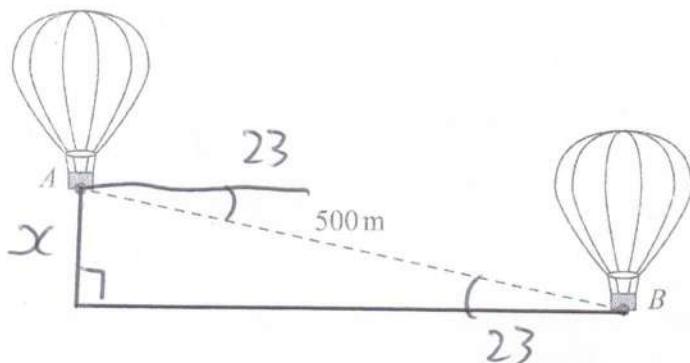


Diagram NOT  
accurately drawn

The distance between  $A$  and  $B$  is 500 metres.

The angle of depression of  $B$  from  $A$  is  $23^\circ$

Calculate the vertical height of  $A$  above  $B$ .

Give your answer correct to one decimal place.

$\text{S} \odot \text{H}$

$$\begin{aligned} x &= \sin 23 \times 500 \\ &= 195.36. \end{aligned}$$

195.4  
metres

(Total for Question 13 is 3 marks)



14 Simon bought a house at the beginning of 2018  
The value of Simon's house had decreased by 15% by the end of 2018

The house increased in value during both 2019 and 2020

The percentage increases in the value of the house during 2019 and 2020 were the same.

The value of Simon's house at the end of 2020 was 2.85% greater than the amount he paid for his house at the beginning of 2018

Calculate the percentage increase in the value of the house during 2019

$$\begin{array}{ccc} 18 & 19 & 20 \\ \curvearrowleft x0.85 & \curvearrowleft xy & \curvearrowleft xy \\ & & \curvearrowright 2.85\% \end{array}$$

$$0.85xy^2 = 1.0285$$

$$y = \sqrt{\frac{1.0285}{0.85}} = 1.10$$

10

%

(Total for Question 14 is 4 marks)



15 Prove algebraically that the product of any two odd numbers is always an odd number.

$$\begin{aligned}& (2n+1)(2m+1) \\&= 4nm + 2m + 2n + 1 \\&= 2(2nm + m + n) + 1 \\&= \text{Even} + 1 \\&\text{hence always odd}\end{aligned}$$

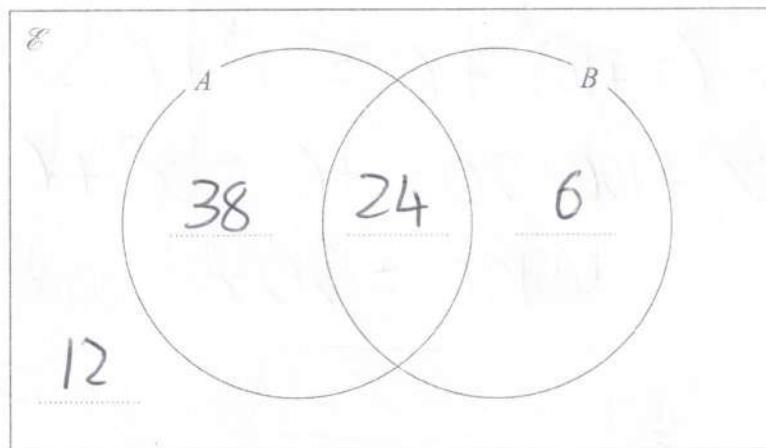
(Total for Question 15 is 4 marks)



16 Two events  $A$  and  $B$  are such that  $n(A) = 62$   $n(B) = 30$  and  $n(A \cup B) = 68$

Given that  $n(\mathcal{E}) = 80$

(a) complete the Venn diagram to show the number of elements in each region.



(2)

An element is chosen at random from  $\mathcal{E}$ .

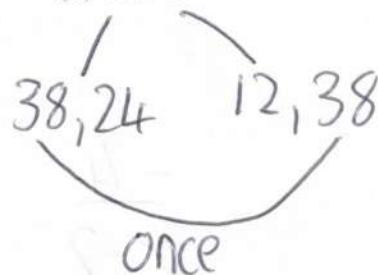
(b) Using the Venn diagram, find the probability that this element is in

(i)  $A \cap B$

$$\frac{24}{80}$$

(1)

(ii)  $A \cup B'$



$$38 + 12 + 24$$

$$\frac{74}{80}$$

(2)

(Total for Question 16 is 5 marks)



17 The functions  $f$  and  $g$  are defined as

$$f(x) = x^2 + 6$$

$$g(x) = x - 10$$

(a) Find  $fg(3)$

$$g(3) = 3 - 10 = -7$$

$$f(-7) = (-7)^2 + 6 = 55$$

(2)

(b) Solve the equation  $fg(x) = f(x)$

Show clear algebraic working.

$$f(x-10) = (x+10)^2 + 6 = x^2 + 6$$

$$\cancel{x^2} + 100 - 20x + 6 = \cancel{x^2} + 6$$

$$100 = 20x$$

$$x = 5$$

(3)

The function  $h$  is defined as

$$h(x) = \frac{2x - 4}{x} = y$$

(c) State the value of  $x$  that cannot be included in the domain of  $h$

$$0$$

(1)

(d) Express the inverse function  $h^{-1}$  in the form  $h^{-1}(x) = \dots$

$$yx = 2x - 4$$

$$yx - 2x = -4$$

$$x(y-2) = -4$$

$$x = \frac{-4}{y-2}$$

$$\frac{-4}{x-2}$$

$$h^{-1}(x) = \dots$$

(3)

(Total for Question 17 is 9 marks)



18 Solve the equation

$$\frac{5}{x+2} + \frac{3}{x^2+2x} = 2$$

Show clear algebraic working.

$$\left[ x \quad x(x+2) \right]$$

$$5x + 3 = 2x^2 + 4x$$

$$0 = 2x^2 - x - 3$$

$$0 = (2x-3)(x+1)$$

$$x = \frac{3}{2}, -1$$

$$x = 1.5, -1$$

(Total for Question 18 is 5 marks)



P 6 6 3 0 2 A 0 1 7 2 4

19 (a) Simplify  $8^2 \times \sqrt[3]{4^6}$

Give your answer in the form  $2^a$  where  $a$  is an integer.  
Show each stage of your working clearly.

$$= (2^3)^2 \times (2^2)^{\frac{6}{3}}$$

$$= 2^6 \times 2^4$$

$$2^{10}$$

(3)

Given that  $n^{\left(-\frac{4}{5}\right)} = \left(\frac{1}{2}\right)^4$  where  $n > 0$

(b) find the value of  $n$ .

$$\begin{aligned} n^{-\frac{4}{5}} &= \left(\frac{1}{2}\right)^4 \\ &= \left(\frac{1}{n}\right)^{\frac{4}{5}} \\ &= \left(\frac{1}{\sqrt[5]{n}}\right)^4 = \left(\frac{1}{2}\right)^4 \end{aligned}$$

$$\sqrt[5]{n} = 2$$

$$n = 2^5$$

$$32$$

$n = \dots$  (4)

(Total for Question 19 is 7 marks)



20  $A$ ,  $B$  and  $C$  are points on a circle with centre  $O$ .

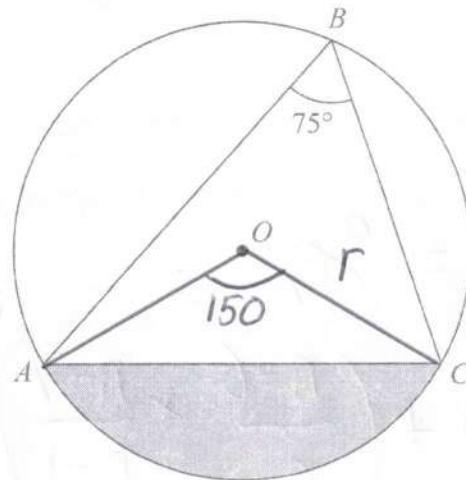


Diagram **NOT**  
accurately drawn

Angle  $ABC = 75^\circ$

The area of the shaded segment is  $200 \text{ cm}^2$

Calculate the radius of the circle.

Give your answer correct to 3 significant figures.

$$200 = \pi r^2 \times \frac{150}{360} - \frac{1}{2} \times r \times r \times \sin 150$$

$$= r^2 \left( \frac{5}{12}\pi - \frac{1}{4} \right)$$

$$r = \sqrt{\frac{200}{\left( \frac{5}{12}\pi - \frac{1}{4} \right)}} = 13.74 \text{ cm}$$

13.7

cm

(Total for Question 20 is 5 marks)



21 A bag contains  $n$  beads.

6 of the beads are red and the rest are blue.

R                    B  
6                     $n-6$

Ravi is going to take at random 2 beads from the bag.

The probability that the 2 beads will be of the same colour is  $\frac{9}{17}$

Using algebra, and showing each stage of your working, calculate the value of  $n$ .

$$RR + BB = \frac{9}{17}$$

$$\left( \frac{6}{n} \times \frac{5}{n-1} \right) + \left( \frac{n-6}{n} \times \frac{n-7}{n-1} \right) = \frac{9}{17}$$

$$\frac{30 + (n-6)(n-7)}{n(n-1)} = \frac{9}{17}$$

$$510 + 17n^2 + 714 - 221n = 9n^2 - 9n$$

$$8n^2 - 212n + 1224 = 0$$

$$2n^2 - 53n + 306 = 0$$

$$n = \frac{53 \pm \sqrt{2809 - 2448}}{4}$$

$\left( n = \frac{17}{2} \text{ is impossible} \right)$

$n = 18$



22  $ABC$  is an isosceles triangle in a horizontal plane. The point  $T$  is vertically above  $B$ .

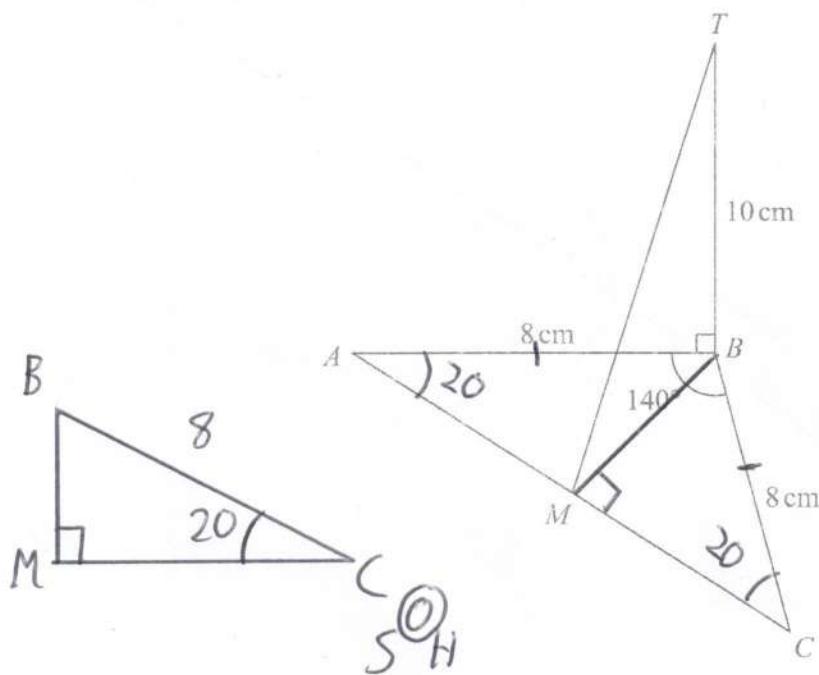


Diagram NOT  
accurately drawn

$$\text{Angle } ABC = 140^\circ$$

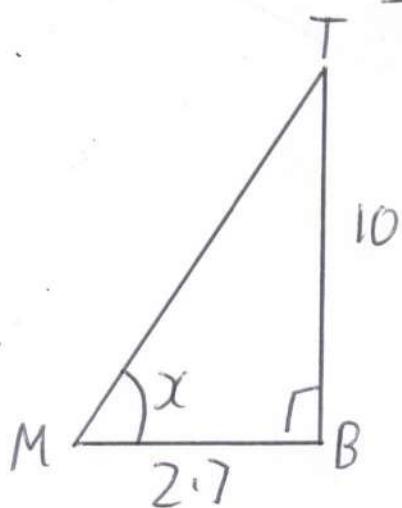
$$AB = BC = 8 \text{ cm}$$

$$TB = 10 \text{ cm}$$

$M$  is the midpoint of  $AC$ .

Calculate the size of the angle between  $MT$  and the horizontal plane  $ABC$ . Give your answer correct to one decimal place.

$$BM = 8 \times \sin 20^\circ = 2.736 \dots$$



DA

$$x = \tan^{-1}\left(\frac{10}{2.7}\right)$$

$$= 74.69 \text{ in}$$

$$> 74.7^\circ$$