Write your name here

Surname		Other names

Centre Number

Candidate Number

Mathematics A

Level 1/2
Paper 2HR

Higher Tier

Thursday 7 June 2018 – Morning
Time: 2 hours

Paper Reference
4MA1/2HR

You must have:
Ruler graduated in centimetres and millimetres, protractor, compasses, pen, HB pencil, eraser, calculator. Tracing paper may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- **Calculators may be used.**
- You must **NOT** write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶

P 5 9 0 1 4 A 0 1 2 4

Pearson

International GCSE Mathematics
Formulae sheet – Higher Tier
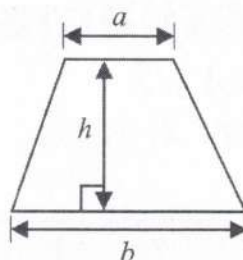
Arithmetic series

Sum to n terms, $S_n = \dfrac{n}{2}\,[2a + (n-1)d]$
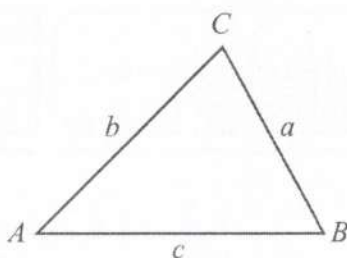
The quadratic equation

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$ are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Area of trapezium $= \dfrac{1}{2}(a + b)h$

Trigonometry
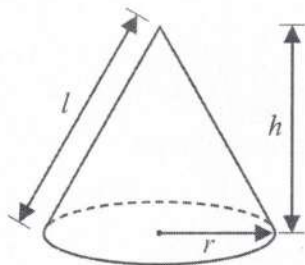

In any triangle ABC

Sine Rule $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine Rule $a^2 = b^2 + c^2 - 2bc \cos A$

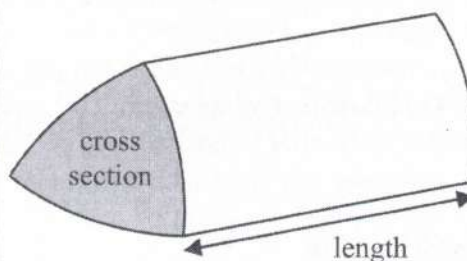**Area of triangle** $= \dfrac{1}{2} ab \sin C$

Volume of cone $= \dfrac{1}{3}\pi r^2 h$

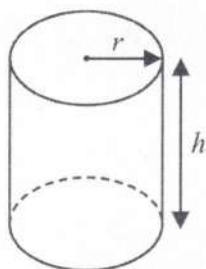**Curved surface area of cone** $= \pi r l$

Volume of prism
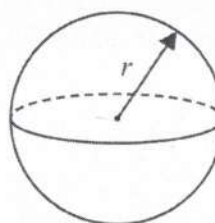= area of cross section × length

Volume of cylinder $= \pi r^2 h$
Curved surface area of cylinder $= 2\pi r h$

Volume of sphere $= \dfrac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$

Answer ALL TWENTY FOUR questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1   x, 10 and y are three integers written in order of size, starting with the smallest integer.

The mean of x, 10 and y is 11 → 33
The range of x, 10 and y is 7

Work out the value of x and the value of y.

$x + 10 + y = 33$

$y + x = 23$

$y - x = 7$ + $y + x = 23$

$2y = 30$

$y = 15$

$15 - x = 7$

$x = 8$

$x =$ _____ 8

$y =$ _____ 15

(Total for Question 1 is 2 marks)

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

2   A box is put on a table.

The face of the box in contact with the table is in the shape of a rectangle, 2 m by 1.25 m.
The pressure on the table due to the box is 42 newtons/m²

Work out the force exerted by the box on the table.

$2 \times 1.25 = 2.5 \, m^2$

$P = \dfrac{F}{A}$ so $F = P \times A$

$= 42 \times 2.5$

105

_____ newtons

(Total for Question 2 is 3 marks)

3 Behnaz makes candles.

$\rightarrow 6300g$

She has 6.3 kilograms of wax and uses it all to make candles.
Each candle Behnaz makes uses 210 grams of wax.

Behnaz sells $\frac{2}{5}$ of the candles for \$13 each.

She then reduces this price by 20% and sells the rest of the candles.

Work out the total amount of money Behnaz gets by selling all the candles she made.

$\frac{6300}{210} = $ 30 candles made

$\frac{2}{5} \times 30 = 12$ $12 \times 13 = \underline{\$156}$

so 18 sold for (13×0.8)

$18 \times 10.4 = \underline{\$187.20}$

Total =

343·20

\$..........

(Total for Question 3 is 4 marks)

4 (a) Expand and simplify $3(c-7) + 2(3c+4)$

$3c - 21 + 6c + 8$

$9c - 13$

(2)

(b) Expand and simplify $(x+7)(x-2)$

$x^2 + 7x - 2x - 14$

$x^2 + 5x - 14$

(2)

(c) Factorise fully $28y^2 - 21y$

$7y(4y - 3)$

(2)

(d) Solve $\dfrac{7x-2}{4} = 3x+1$

Show clear algebraic working.

$7x - 2 = 12x + 4$

$-6 = 5x$

$\dfrac{-6}{5} = x$

$x = -\dfrac{6}{5}$

(3)

(Total for Question 4 is 9 marks)

5 Abelie flew by plane from Dubai to Rome.

$\rightarrow 6\frac{42}{60}$ hr

$= 6.7$ hr

The flight time was 6 hours 42 minutes.
The average speed of the plane was 650 kilometres per hour.

Work out the distance the plane flew.

$$D = 650 \times 6.7$$

...........................4355...........................kilometres

(Total for Question 5 is 3 marks)

6 Hiran invests 20 000 rupees in an account for 3 years at 1.5% per year compound interest.

Work out the total amount of money in the account at the end of 3 years.
Give your answer to the nearest rupee.

$$20000 \times 1.015^{3}$$

$$= 20913.5675$$

...........................20914...........................rupees

(Total for Question 6 is 3 marks)

7 (a) Simplify fully $\dfrac{20x^2y^6}{4x^2y^2}$

$20 \div 4 = 5$ $2-2=0$

$6-2=4$

$\underline{5y^4}$

(2)

(b) Make e the subject of the formula $h = 3e + f$
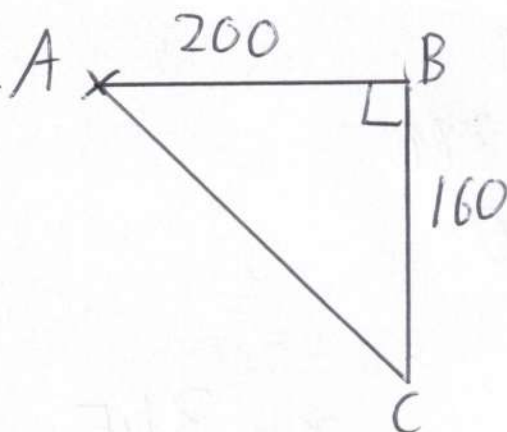
$h - f = 3e$

$e = \dfrac{h-f}{3}$

(2)

(Total for Question 7 is 4 marks)

8 From point A, Stanley walks 200 m due east to point B.
From B, he then walks 160 m due south to point C.

Work out the length of AC.
Give your answer correct to 3 significant figures.

$AC = \sqrt{200^2 + 160^2}$

$= 40\sqrt{41}$

$= 256 \cdot 1249...$

$A \times \underline{\quad 200 \quad} B$

160

C

$\underline{256}$ ....metres

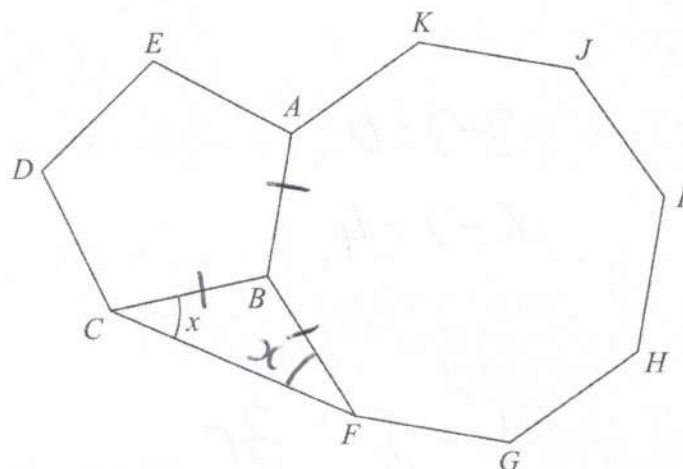**(Total for Question 8 is 3 marks)**

P 5 9 0 1 4 A 0 7 2 4

Diagram **NOT** accurately drawn

The diagram shows a regular pentagon, *ABCDE*, a regular octagon, *ABFGHIJK*, and an isosceles triangle, *BCF*.

Work out the size of angle *x*.

Pentagon internal $= \dfrac{(5-2) \times 180}{5} = 108$

Octagon int $= \dfrac{(8-2) \times 180}{8} = 135$

$\angle CBF = 360 - 135 - 108$
$\quad = 117$

$\triangle CBF : 2x + 117 = 180$
$\quad 2x \quad = 63$

$x = 31.5$ °

(Total for Question 9 is 4 marks)

10 *ABCD* is a trapezium.

Diagram **NOT** accurately drawn

B ———16 cm——— C

12.5 cm

A ———24.3 cm——— D

x y
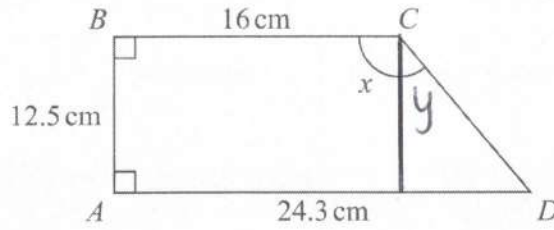
Work out the size of angle x.
Give your answer correct to 1 decimal place.

12.5 y

(24.3 − 16
= 8.3)
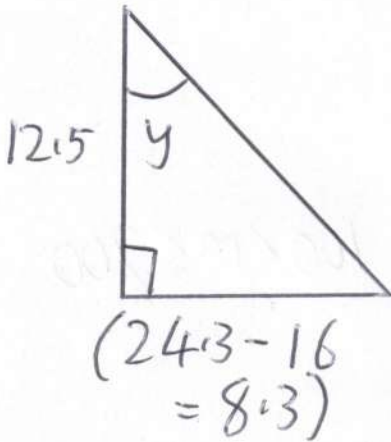
$$y = \tan^{-1}\left(\frac{8.3}{12.5}\right)$$

$$= 33.5841...$$

$$x = y + 90 = 123.6$$ °

11 The table shows information about the amount of money spent on holiday by each of 120 families.

Money spent (£m)	Frequency
$0 < m \leqslant 100$	10
$100 < m \leqslant 200$	36
$200 < m \leqslant 300$	34
$300 < m \leqslant 400$	20
$400 < m \leqslant 500$	15
$500 < m \leqslant 600$	5

(a) Write down the modal class.

$100 < m \leqslant 200$

(1)

(b) Complete the cumulative frequency table for the information in the table.

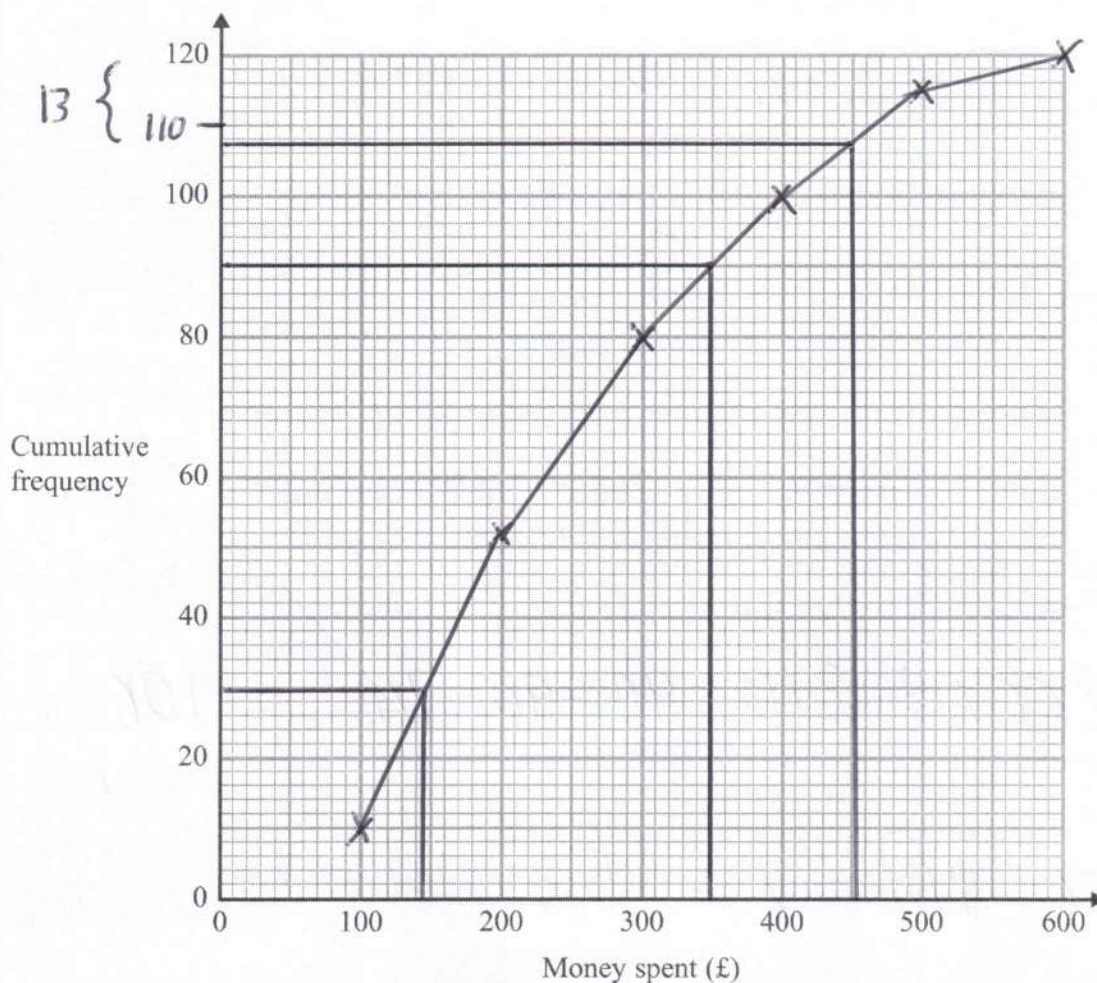| Money spent (£m)	Cumulative frequency
$0 < m \leqslant 100$	10
$0 < m \leqslant 200$	46
$0 < m \leqslant 300$	80
$0 < m \leqslant 400$	100
$0 < m \leqslant 500$	115
$0 < m \leqslant 600$	120

(1)

(c) On the grid, draw a cumulative frequency graph for your table.

(2)

13 { 110

Cumulative frequency

Money spent (£)

(d) Use your graph to find an estimate for the interquartile range.

350 − 145

[ms : 175 − 205]

£ 205

(2)

(e) Use your graph to find an estimate for the number of families that spent more than £450 on holiday.

[ms: 12 or 13]

13

(2)
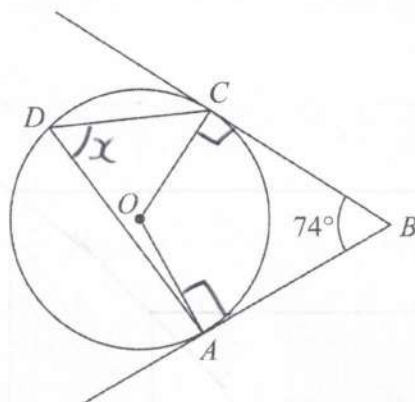
(Total for Question 11 is 8 marks)

P 5 9 0 1 4 A 0 1 1 2 4

Diagram **NOT** accurately drawn

A, C and D are points on a circle, centre O.
AB and CB are tangents to the circle.

Angle $ABC = 74°$

Work out the size of angle ADC.
Show your working clearly.

$\angle COA = 360 - 90 - 90 - 74 = 106$

$x = 106 \div 2 = 53$

53 °

(Total for Question 12 is 3 marks)

P 5 9 0 1 4 A 0 1 2 2 4

13 The straight line L_1 has equation $y = 6 - 2x$

The straight line L_2 is perpendicular to L_1 and passes through the point (4, 7)

Find the coordinates of the point where the line L_2 crosses the x-axis.

$L_1 : m = -2$

$L_2 : m = \frac{1}{2}$

$y = mx + c$

$7 = \frac{1}{2} \times 4 + c$

$5 = c$

so $y = \frac{1}{2}x + 5$

crosses x-axis   when $y = 0$

so $0 = \frac{1}{2}x + 5$

$\frac{1}{2}x = -5$

(......-10......, ...0.....)

(Total for Question 13 is 4 marks)

14 $128 = 4^{2x} \times 2^x$

Work out the value of x.

$2^7 = \left(2^2\right)^{2x} \times 2^x$

$2^7 = 2^{4x} \times 2^x$

so $7 = 4x + x$

$5x = 7$

$x = $ ....$\frac{7}{5}$ or 1.4....

(Total for Question 14 is 3 marks)

P 5 9 0 1 4 A 0 1 3 2 4

15 The Venn diagram shows a universal set, $\mathscr{E}$, and sets A, B and C.
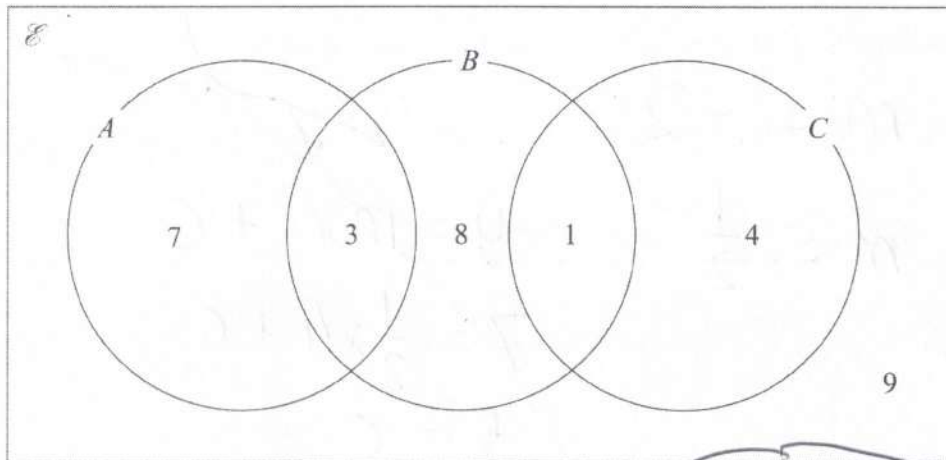
7, 3, 8, 1, 4 and 9 represent the **numbers** of elements.

strange!

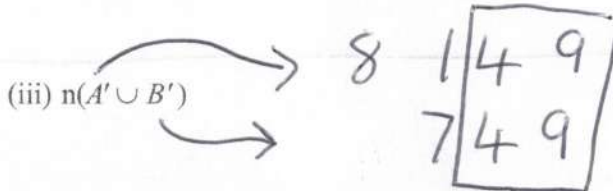Find

(i) $n(A \cup B)$

$$7 + 3 + 8 + 1 = 19$$

(ii) $n(A' \cap C)$
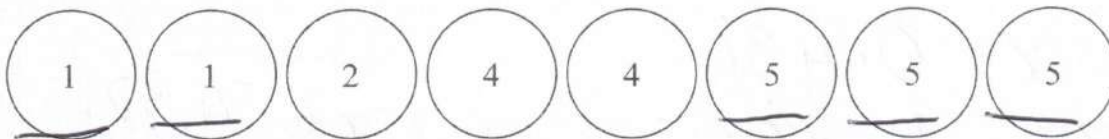
$$1 + 4 = 5$$

(iii) $n(A' \cup B')$

8   1 | 4   9
7 | 4   9

29

(Total for Question 15 is 3 marks)

only count each ONCE

so   8 + 1 + 7 + 4 + 9

16 There are 8 counters in a bag.
There is a number on each counter.

$$\boxed{1} \quad \boxed{1} \quad \boxed{2} \quad \boxed{4} \quad \boxed{4} \quad \boxed{5} \quad \boxed{5} \quad \boxed{5}$$

Fiona takes at random **three** of the counters.
She adds the numbers on the **three** counters to get her total.

Work out the probability that her total is an odd number.

$Odd = OOO \quad or \quad \begin{array}{l} EEO \\ EOE \\ OEE \end{array}$

$$= \left(\frac{5}{8} \times \frac{4}{7} \times \frac{3}{6}\right) + 3 \times \left(\frac{3}{8} \times \frac{2}{7} \times \frac{5}{6}\right)$$

$$= \frac{60}{336} + \frac{90}{336}$$

$$\frac{25}{56}$$

17 (a) Use algebra to show that $0.4\overset{..}{3}\overset{..}{6} = \dfrac{24}{55}$

$$x = 0.4\overset{..}{3}\overset{..}{6}$$
$$10x = 4.\overset{..}{3}\overset{..}{6}$$
$$1000x = 436.\overset{..}{3}\overset{..}{6}$$

$$\overline{990x = 432}$$

$$x = \frac{432}{990}$$

$$x = \frac{24}{55}$$

(2)

(b) Show that $\dfrac{\sqrt{20} + \sqrt{80}}{\sqrt{3}}$ can be expressed in the form $\sqrt{a}$ where a is an integer.

Show your working clearly.

$$\frac{\sqrt{4}\sqrt{5} + \sqrt{16}\sqrt{5}}{\sqrt{3}}$$

$$= \frac{2\sqrt{5} + 4\sqrt{5}}{\sqrt{3}}$$

$$= \frac{6\sqrt{5}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{15}}{3} = 2\sqrt{15}$$
$$= \sqrt{4}\sqrt{15}$$
$$= \underline{\sqrt{60}}$$

(3)

(Total for Question 17 is 5 marks)

18 Solve the simultaneous equations

$$2x^2 + 3y^2 = 14$$
$$x = 2y - 3$$

Show clear algebraic working.

$$2(2y-3)^2 + 3y^2 = 14$$

$$8y^2 + 18 - 24y + 3y^2 = 14$$

$$11y^2 - 24y + 4 = 0$$

$$(11y - 2)(y - 2) = 0$$

$$y = 2 \qquad\qquad y = \frac{2}{11}$$

$$x = 2 \times 2 - 3 = 1 \qquad\qquad x = 2 \times \frac{2}{11} - 3$$

$$= -\frac{29}{11}$$

$$x = 1, y = 2 \quad \Big| \quad x = -\frac{29}{11}, y = \frac{2}{11}$$

(Total for Question 18 is 5 marks)

19 $a = \dfrac{p - q}{t}$

$p = 8.4$ correct to 2 significant figures.
$q = 6.3$ correct to 2 significant figures.
$t = 0.27$ correct to 2 significant figures.

Work out the upper bound for the value of a.
Show your working clearly.
Give your answer correct to 1 decimal place.

$$8.4 < \begin{array}{c} 8.45 \\ 8.35 \end{array} \qquad 6.3 < \begin{array}{c} 6.35 \\ 6.25 \end{array} \qquad 0.27 < \begin{array}{c} 0.275 \\ 0.265 \end{array}$$

$$UB = \frac{\uparrow p - \downarrow q}{\downarrow t}$$

$$= \frac{8.45 - 6.25}{0.265}$$

$$= 8.3018\ldots$$

8.3
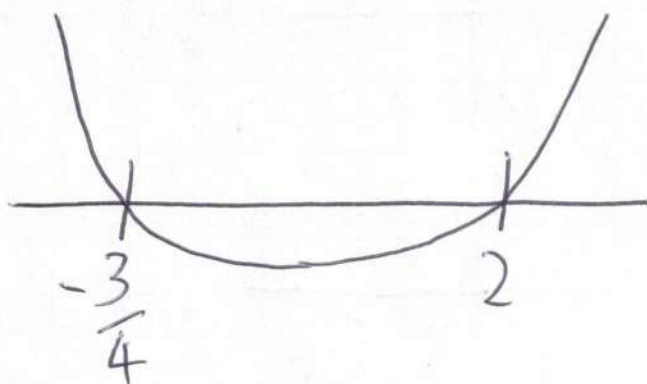
(Total for Question 19 is 3 marks)

20 Solve the inequality $4x^2 - 5x - 6 > 0$

$$(4x+3)(x-2) > 0$$

> 0

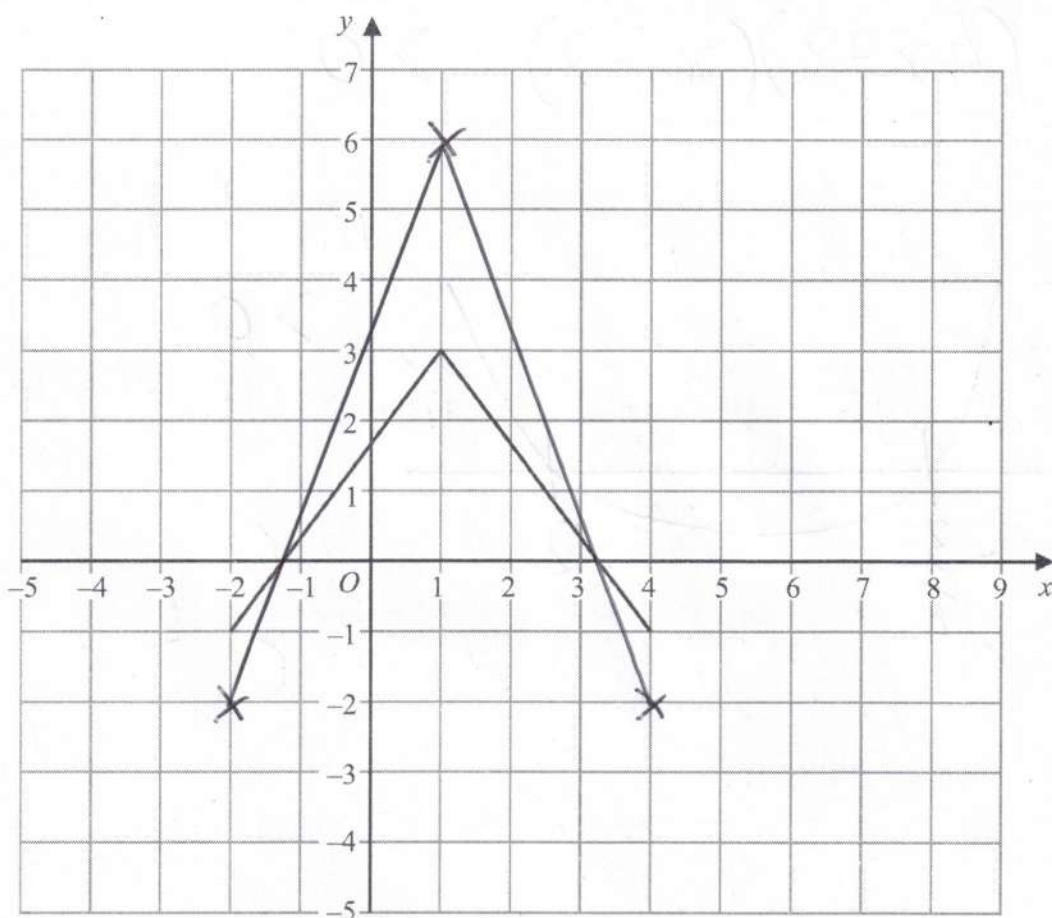$$x < -\frac{3}{4} \qquad x > 2$$

(Total for Question 20 is 4 marks)

21 Here is the graph of $y = f(x)$

(a) On the grid above, draw the graph of $y = 2f(x)$

(2)
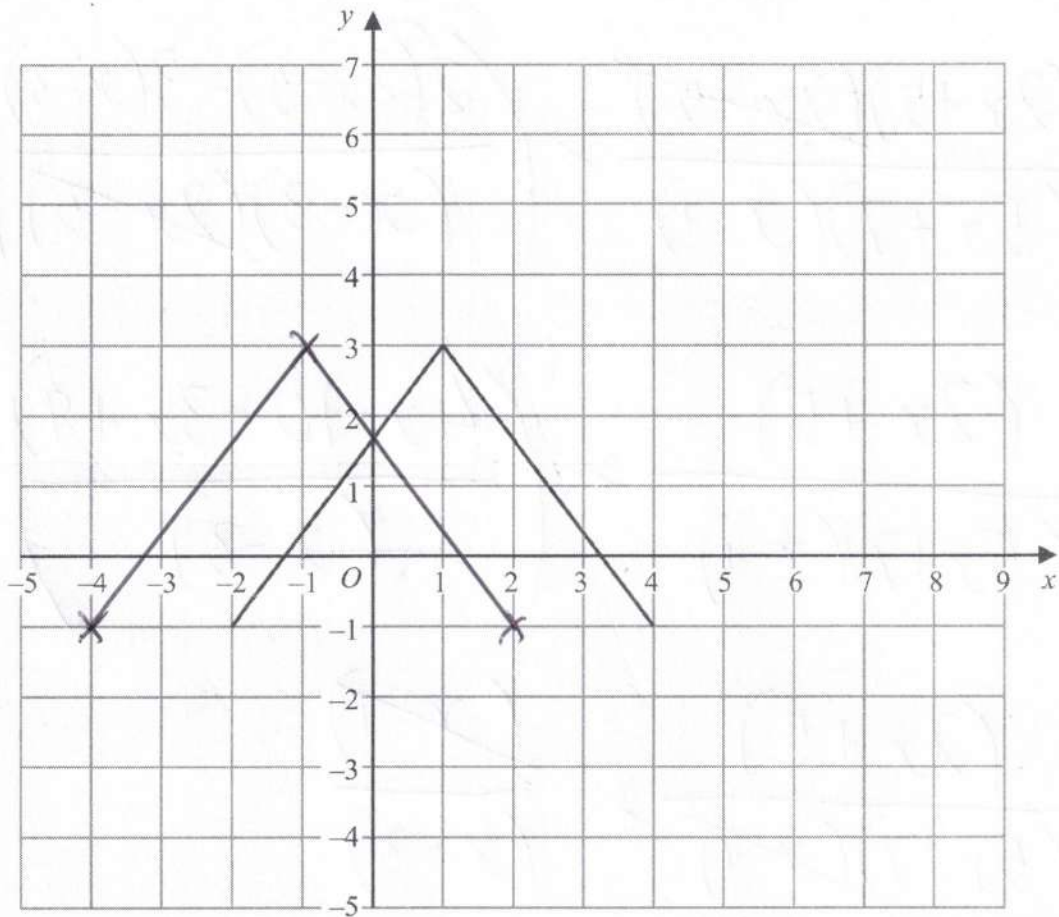
scale factor $\updownarrow \times 2$

Here is the graph of $y = f(x)$

(b) On the grid above, draw the graph of $y = f(-x)$

reflect in x-axis

(2)

(Total for Question 21 is 4 marks)

22 Express $\dfrac{4x^2 - 25}{5x^2 + 2x - 7} \times \left(\dfrac{2}{x-3} - \dfrac{3}{2x-5} \right)$ as a single fraction in its simplest form.

$$\frac{(2x+5)\cancel{(2x-5)}}{(5x+7)(x-1)} \times \left(\frac{2(2x-5) - 3(x-3)}{(x-3)\cancel{(2x-5)}} \right)$$

$$\frac{(2x+5)}{(5x+7)(x-1)} \times \left(\frac{4x-10-3x+9}{(x-3)} \right)$$

$$\frac{(2x+5)}{(5x+7)\cancel{(x-1)}} \times \frac{\cancel{(x-1)}}{(x-3)}$$

$$\frac{2x+5}{(5x+7)(x-3)}$$

P 5 9 0 1 4 A 0 2 2 2 4
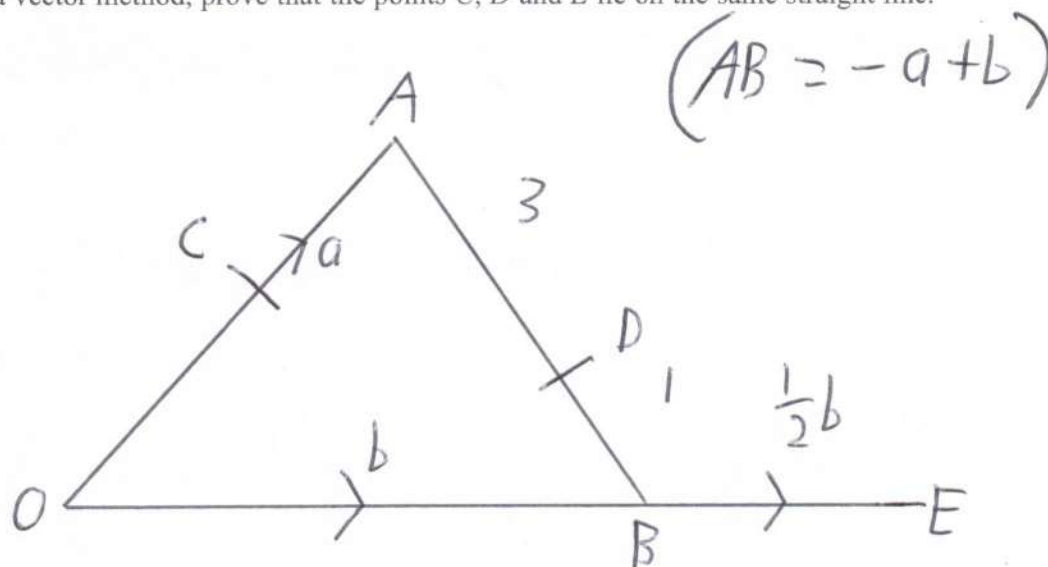
23 OAB is a triangle.

$\overrightarrow{OA} = \mathbf{a}$ $\overrightarrow{OB} = \mathbf{b}$

C is the midpoint of OA.
D is the point on AB such that $AD:DB = 3:1$
E is the point such that $\overrightarrow{OB} = 2\overrightarrow{BE}$

Using a vector method, prove that the points C, D and E lie on the same straight line.

$$\left(AB = -a + b \right)$$

$$\overrightarrow{CD} = \overrightarrow{CA} + \overrightarrow{AD}$$
$$= \frac{1}{2}a + \frac{3}{4}\overrightarrow{AB}$$
$$= \frac{1}{2}a + \frac{3}{4}b - \frac{3}{4}a$$
$$= -\frac{1}{4}a + \frac{3}{4}b$$
$$= \frac{1}{4}(-a + 3b)$$

$$\overrightarrow{CE} = \overrightarrow{CO} + \overrightarrow{OE}$$
$$= -\frac{1}{2}a + \frac{3}{2}b$$
$$= \frac{1}{2}(-a + 3b)$$

$$\overrightarrow{CD} \times 2 = \overrightarrow{CE}$$

hence   straight line

(Total for Question 23 is 5 marks)

24 (a) Express $7 - 4x - x^2$ in the form $p - (x + q)^2$ where p and q are constants.

$$-\left[x^2 + 4x - 7\right]$$
$$-\left[(x+2)^2 - 4 - 7\right]$$
$$-\left[(x+2)^2 - 11\right]$$
$$-(x+2)^2 + 11$$

$$11 - (x+2)^2$$ ....

(2)

(b) Use your answer to part (a) to solve the equation $7 - 4(y + 3) - (y + 3)^2 = 0$

Give your solutions in the form $e \pm \sqrt{f}$ where e and f are integers.

$$\boxed{\text{So} \quad x = y + 3}$$
$$7 - 4x - x^2 = 0$$
$$\Rightarrow \quad 11 - (x + 2)^2 = 0$$
$$x + 2 = \pm\sqrt{11}$$
$$x = -2 \pm \sqrt{11}$$
$$y + 3 = -2 \pm \sqrt{11}$$

$$y = -5 \pm \sqrt{11}$$ ....

(3)

The curve **C** has equation $y = 3 - 5(x + 1)^2$
The point A is the maximum point on **C**.

(c) Write down the coordinates of A.

(...........–1........... , ...3...........)

(1)

(Total for Question 24 is 6 marks)

TOTAL FOR PAPER IS 100 MARKS